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Controllability and Stabilization of Parabolic Equations

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Preface

This book is based on the author's works and lectures on controllability and stabilization of parabolic equations and part of it was used for a graduate course at the University of Iaşi, Romania.

In the inaugural lecture delivered at Warwick on the 7th of October 1970, Lawrence Markus has briefly defined the objectives of the mathematical control theory as:

> "... *the modification of differential equations, within prescribed limitations so that the solutions behave in some desired manner. The common feature of all these problems is that we prescribe the desired behaviour of the solutions and then we seek to modify the coefficients of the dynamical equations so as to induce this behaviour. In classical mathematical physics, we know all about the physical laws guiding the development of the phenomenon we may be studying – we do not know all the rules of the game – and we want to predict the outcome. In control theory, we know the rules, in fact we can change them within certain limitations, but we do know exactly how we want the game to end. Thus, the mathematical problems of control theory are inverse to the usual problems of mathematical physics.*"

The controllability and stabilization are without any doubt two fundamental problems of mathematical control theory which, for parabolic-like systems, have a special significance and difficulty due to the time irreversibility of dynamics generated by these systems.

The first part of the book is devoted to the internal null controllability of parabolic equations based essentially on the influential linear controllability result of G. Lebeau and L. Robbiano (1995), developed later on by A.V. Fursikov and O.Yu. Imanuvilov (1996), via a Carleman-type inequality for linear parabolic equations. Compared with controllability, internal and boundary stabilization are apparently weaker properties but, since they are usually realized by feedback controllers, they are structurally stable and so more convenient for applications and numerical simulations. The first stabilization method developed here is based on the spectral decomposition technique, which is used to represent the linear parabolic control systems in a convenient product space as a null controllable finite dimensional unstable system and a stable infinite dimensional one. Such a system is stabilizable by an open loop controller, and so a stabilizable linear feedback controller can be obtained by standard optimal control arguments from an algebraic Riccati equation. As a matter of fact, this stabilization method applies to general semilinear control systems of the form $\frac{dy}{dt} + Ay + Fy = Bu$ in a Hilbert space H, where the operator $-A$ is the infinitesimal generator of a C_0-analytic semigroup in H and has a compact resolvent. This is the class of so-called parabolic-like systems which, besides the standard heat and diffusion equations, includes Navier–Stokes and other related systems such as Boussinesq and magnetohydrodynamic equations.

The second method mostly applicable to boundary control systems consists in the explicit construction of a stabilizing feedback controller by using a finite number of unstable modes. Both methods provide stabilizable feedback controllers with finite dimensional structure and stabilization. The exact controllability of stochastic parabolic equations with linear multiplicative noise is also briefly treated in this book, though so far only partial results were obtained in this direction.

It should be mentioned, however, that none of the above topics was exhaustively treated in this book, which may be viewed only as a survey on controllability and stabilization techniques for parabolic boundary value problems. Many of these results, exact controllability in particular, keep their original strength and flavor, even though they were established at the end of the 1990s.

There are several important topics related to the content of this book which were not treated here and most notable is perhaps the controllability of Navier–Stokes equations extensively studied in the last two decades.

We wish to thank J.M. Coron for the invitation to write this book and publish it with the *Control/PNLDE* series.

Thanks are due also to my colleagues Gabriela Marinoschi, Cătălin-George Lefter and Ionuţ Munteanu who read and criticized various chapters.

Thanks are also due to the anonymous reviewers for their suggestions.

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Iasi, Romania Viorel Barbu
January 1st, 2018

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Acronyms

p_K	the Minkowski functional of the set K
$\mathrm{recc}(K)$	the recession cone of K
$\mathbb{C}$	the set of all complex numbers
$\mathbb{N}$	the set of all natural numbers
$\mathbb{R}$	the real line $(-\infty, \infty)$
$\mathbb{R}^d$	the d-dimensional Euclidean space
$\mathbb{R}^+$	$= (0, +\infty)$
$\mathbb{R}^d_+$	$= \{(x_1, \dots, x_d);\ x_d > 0\}$
$\mathscr{O}$	an open subset of $\mathbb{R}^d$
$\partial\mathscr{O}$	the boundary of $\mathscr{O}$
Q	$= \mathscr{O} \times (0, T)$
Σ	$= \partial\mathscr{O} \times (0, T)$, where $0 < T < \infty$
$\Vert \cdot \Vert_X$	the norm of the linear normed space X
X^*	the dual of the space X
$(\cdot, \cdot)_H$	the scalar product of the Hilbert space H
$x \cdot y$	the scalar product of the vectors $x, y \in \mathbb{R}^d$
$L(X, Y)$	the space of linear continuous operators from X to Y
∇f	the gradient of the function f
∂f	the subdifferential of the function f
B^*	the adjoint of the operator B
$\overline{C}$	the closure of the set C
$\mathrm{int}\, C$	the interior of the set C
$\mathrm{conv}\, C$	the convex hull of the set C
$D(A)$	the domain of the operator A
$R(A)$	the range of the operator A
I_C	the indicator function of the set C
sign	the signum function on X : $\mathrm{sign}\, x = x/\Vert x\Vert_X$ if $x \neq 0$, $\mathrm{sign}\, 0 = \{x;\ \Vert x\Vert_X \leq 1\}$
$C^k(\mathscr{O})$	the space of real-valued functions on $\mathscr{O}$ that are continuously differentiable up to order k, $k \leq \infty$

$C_0^k(\mathscr{O})$	the subspace of functions in $C^k(\mathscr{O})$ with compact support in $\mathscr{O}$
$\mathscr{D}(\mathscr{O})$	the space $C_0^\infty(\mathscr{O})$
$\frac{d^k u}{dt^k}, u^{(k)}$	the derivative of order k of the function $u : [a, b] \to X$
$\mathscr{D}'(\mathscr{O})$	the dual of $\mathscr{D}(\mathscr{O})$ (i.e., the space of distributions on $\mathscr{O}$)
$C(\overline{\mathscr{O}})$	the space of continuous functions on $\overline{\mathscr{O}}$
$L^p(\mathscr{O})$	the space of p-summable functions $u : \mathscr{O} \to \mathbb{R}$ endowed with the norm $\|u\|_p = (\int_{\mathscr{O}} \|u(x)\|^p dx)^{\frac{1}{p}}$, $1 \le p < \infty$
$L_m^p(\mathscr{O})$	the space of p-summable functions $u : \mathscr{O} \to \mathbb{R}^m$
$W^{m,p}(\mathscr{O})$	the Sobolev space $\{u \in L^p(\mathscr{O});\ D^\alpha u \in L^p(\mathscr{O}), \|\alpha\| \le m,\ 1 \le p \le \infty\}$
$W_0^{m,p}(\mathscr{O})$	the closure of $C_0^\infty(\mathscr{O})$ in the norm of $W^{m,p}(\mathscr{O})$
$W^{-m,q}(\mathscr{O})$	the dual of $W_0^{m,p}(\mathscr{O})$; $\frac{1}{p} + \frac{1}{q} = 1,\ p < \infty,\ q > 1$
$H^k(\mathscr{O}), H_0^k(\mathscr{O})$	the spaces $W^{k,2}(\mathscr{O})$ and $W_0^{k,2}(\mathscr{O})$, respectively
$L^p(a, b; X)$	the space of p-summable functions from (a, b) to X, $1 \le p \le \infty,\ -\infty \le a < b \le \infty$
$C([a, b]; X)$	the space of X-valued continuous functions on $[a, b]$
$AC([a, b]; X)$	the space of absolutely continuous functions from $[a, b]$ to X
$W^{1,p}([a, b]; X)$	the Sobolev space $\left\{u \in AC([a, b]; X); \frac{du}{dt} \in L^p((a, b); X)\right\}$
ν, n	the outward normal to $\mathscr{O}$
$\frac{\partial u}{\partial \nu}, \frac{\partial u}{\partial n}$	the normal derivative of the function $u : \mathscr{O} \to \mathbb{R}$