Conformal Geometry

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Computational Algorithms and Engineering Applications

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For those who love geometry.

Preface

Conformal means angle preserving in mathematics. *Conformal geometry* studies *conformal structure* of general surfaces. Conformal structure is a natural geometric structure and a special atlas on surfaces such that angles among tangent vectors can be coherently defined on different local coordinate systems. Conformal structure governs many physics phenomena including heat diffusion and electric–magnetic fields.

Computational conformal geometry focuses on algorithmic study of conformal geometry and offers powerful tools to handle a broad range of geometric problems in engineering fields. It links modern geometry theories to real engineering applications.

The power of computational conformal geometry in engineering fields stems from the following fundamental reasons.

- Conformal geometry studies surface conformal structure. All surfaces in daily life have a natural conformal structure. Therefore, geometric algorithms based on conformal geometry benefit general surfaces.
- Conformal structure of a general surface is more flexible than Riemannian metric structure and more rigid than topological structure. It can handle large deformations, which Riemannian geometry cannot efficiently handle; it preserves a lot of geometric information during the deformation, whereas topological methods lose too much information.
- Conformal maps are easy to control. For example, the conformal maps between two simply-connected closed surfaces form a six-dimensional space; therefore by fixing three points, the mapping is uniquely determined. This fact makes conformal geometric method very valuable for surface matching and comparison.
- Conformal maps preserve local shapes; therefore, it is convenient for visualization purposes.
- All surfaces can be classified according to their conformal structures, and all the conformal equivalent classes form a finite-dimensional manifold. This manifold has rich geometric structures and can be analyzed and studied. In comparison,

the isometric classes of surfaces form an infinite-dimensional space, and it is really difficult to deal with.

- Computational conformal geometric algorithms are based on solving elliptic partial differential equations, which are easy to solve, and the solving process is stable; namely, the solution is insensitive to the noise of the input surfaces. Therefore, computational conformal geometry method is very practical for real engineering applications.
- In conformal geometry, all surfaces in daily life can be deformed to three canonical spaces, the sphere, the plane, or the disk (the hyperbolic space). In other words, any surface admits one of the three canonical geometries, spherical geometry, Euclidean geometry, or the hyperbolic geometry. Most digital geometric processing tasks in three-dimensional space can be converted to the task in these two-dimensional canonical spaces.

The book provides an overview of computational conformal geometry applied in engineering fields. We first briefly introduce the major concepts and theorems of conformal geometry in an intuitive way with a large number of illustrative images rendered by graphics tools.

In part I of the book, we detail the major computational algorithms in conformal geometry in an accessible way for computer scientists and engineers. We provide less abstract mathematical reasoning, but more intuitive explanations and implementation issues from the engineering point of view.

In part II of the book, we dedicate each chapter to a specific application field of computational conformal geometry including computer graphics, computer vision, geometric modeling, medical imaging, and wireless sensor networks. We discuss the fundamental problems, and how computational conformal geometric methods tackle them in a theoretically elegant and computationally efficient way in each field.

Computational conformal geometry is an emerging field. There are still a lot of challenging and open problems both in theory and in practice. Applying computational conformal geometric methods to broader applications and adapting them to real systems are still developing. The book will be of interest to senior undergraduates, graduates, and researchers in computer science, applied mathematics, and wide branches of engineering.

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