

ICME-13 Monographs

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Research on Mathematics Textbooks and Teachers' Resources

Advances and Issues

 Springer

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Luc Trouche is Professor of Didactics of Mathematics in the French Institute of Education, École normale supérieure de Lyon, France.

In previous years, his research was dedicated to studying ICT integration in Mathematics Education. In particular, he has studied the interplay between *instrumentation* processes and *conceptualization* processes. This work has led him to analyse the teacher's role, introducing the notion of *instrumental orchestration* for modelling the management of available artefacts (for teaching a particular mathematical topic) in the classroom. He focuses now on resource design and teacher professional development in the time of digitalization. This has led him to develop, in a joint work with Ghislaine Gueudet and Birgit Pepin, the *documentational approach to didactics*. In this perspective, the notion of teacher *resource system* appears crucial in order to understand teacher's (developing) knowledge and the coherence of his/her activity. Investigating teachers' work with

resources leads to consider teachers' collective work. Indeed, digitalization favours the emergence of more or less formal collectives: networks; teaching groups; and learning communities within or out of schools. Studying the interactions between individual and collective teachers' resource systems gives means for understanding the dynamics of these collectives, and for rethinking the way of supporting teacher development at a time of the "metamorphosis" of teaching environments.

He is currently involved in collaborative research at an international level, particularly with China, Brazil, or Mexico. He is the Chair of the International Conference Re(s)ources 2018, to be held in France in 2018. Among his publications: *From Text to 'Lived' Resources: Mathematics Curriculum Materials and Teacher Development*, co-edited with Birgit Pepin, and Ghislaine Guedet (Springer, 2012) and *Tools and Mathematics: Instruments for Learning*, co-written with John Monaghan and Jon Borwein (Springer, 2016).

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Jana Visnovska is a Lecturer in Mathematics Education at the University of Queensland, Australia. Her research interests focus on the means of supporting mathematics teachers that empower them in providing all students with opportunities to learn meaningful mathematics. To this end, she explores features of teachers' resources and forms of professional development that are in particular effective in supporting the learning and instructional interactions of teachers in transition. In ongoing research collaboration with Jose Luis Cortina, she contributes to instructional design research in area of fractions as measures and has been intrigued by the role of quantity in school mathematics education. Through her research, she seeks opportunities to learn about mathematics teaching and learning in different educational contexts, including severely

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Mathematics Textbooks and Teachers' Resources: A Broad Area of Research in Mathematics Education to be Developed

Luc Trouche and Lianghuo Fan

Abstract

This introduction aims to situate this monograph as a result of the work of the Topic Study Group (TSG) 38 at the 13th International Congress on Mathematical Education, held in Hamburg in July 2016. Beyond this goal, it aims to evidence the fact that, far from being an isolated work, the contributions received to this TSG, their number as well as their content, witness the emergence, at an international level, of a new field of research, dedicated to the teachers' resources. After having set this scene, this introduction shortly overviews the chapters of this monograph, structured in five parts.

Keywords

Mathematics teachers' resources • Learning materials • Textbooks • Mathematics education • ICME-13

The Origins of this Monograph

This monograph originates in the work of the 38th Topic Study Group (TSG 38) of the 13th International Congress on Mathematical Education (ICME-13), held in

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Hamburg, Germany in July 2016. TSG 38 was dedicated to *Research on resources (textbooks, learning materials, etc.)*, and was intentionally broadly inclusive of school mathematics textbooks, all forms of teacher manuals, learning and assessment materials, and in particular of digital and online resources.

One of the aims of TSG 38 was to bring to the foreground and examine various theoretical and methodological approaches that were being used to study teaching and learning resources. In relation to this, the TSG 38 call for papers sought contributions that addressed broadly the areas of resources, teachers, and students, and in particular encouraged contributions that presented analyses of the evolution of interactions between resources, teachers, and students in a time of transition.

To further guide and elicit the richness of contributions and to shape the future work of TSG 38, the call included following questions about the resources:

- Among the teaching and learning materials available in mathematics classrooms in different countries, what role do textbooks and other curricular or learning resources play in mathematics teaching, learning, and assessment?
- Could we consider different types of textbooks which suit for project-based, inquiry-based, or problem-based learning?
- How does the digitalization of information and communication affect the role of textbooks?
- What are defining features of e-textbooks, and how could we characterize the differences between the traditional textbooks and e-textbooks?

Guiding questions that focused on teachers' role included:

- What are the relationships between teachers' individual and collective resources, and how could we model such relationships?
- What is the role of teachers in developing textbooks and other teaching and learning materials?
- What are the relationships between resource designers and users?
- What are the consequences of evolutions at stake for the teaching of mathematics, and for teacher's knowledge and professional development?

Finally, guiding questions that focused on students' interactions with learning resources included:

- How do students, as well as their teachers, interact through resources?
- What are the effects of modern ICT (particularly Internet) on students' use and the design of resources?
- How do these evolutions affect student behaviour, learning, and relationships to the subject of mathematics?

(See also http://www.icme13.org/files/tsg/TSG_38.pdf for the call for papers.)

The organizers of TSG 38—editors of this monograph—received about 80 answers to this call. Sixteen of them were retained as long contributions to four plenary sessions, each dedicated to one of the following topics:

- (1) What role do textbooks and other curricular or learning resources play in mathematics teaching, learning, and assessment?
- (2) How does the digitalization of information and communication affect the role of resources?
- (3) Teachers' collective work through resources;
- (4) Teachers' and students' interactions through resources.

Out of the submissions received, 45 were retained as short oral contributions, which were presented in several chaired parallel sessions. Far from bringing copies of the papers proposed to TSG 38 before ICME-13, this monograph is the result of a process that began with the TSG 38 call, continued with the rich discussions arising in each TSG session, plenary as well as parallel sessions, and concluded with the review and editing work on 15 selected papers, chosen for their quality, but also for their emblematic character in the field of research. Speaking of a single field of research may appear as a simplification, as the discussion about teachers' resources gathered researchers with various major interests, inevitably including textbooks, as well as teachers' digital resources, curriculum resources, ICT, task design, Internet usage, teachers' communities of practice, and teachers' professional development. The organization of this TSG was based on a main assumption: the evolution of textbooks (to e-textbooks), the dissemination, by various institutions, of curricular resources through the means of the Internet, and the emergence of teachers' communities designing and sharing their own resources, all evidence the need for a new field of research that this book aims to explore and pursue.

A New Field of Research

The emergence of the new field of research related to teachers' resources is noticeable in both the theme-specific spaces appearing within the existing scientific events of the mathematics education community, and the growth of new, dedicated scientific events with this focus.

In a sense, ICME-13 was the first of the major four-yearly international congresses of mathematics educators to dedicate a TSG to the theme of teachers' resources. In past, contributions related to this theme were scattered across various TSGs. For example, four years earlier, ICME-12 (<http://www.icme12.org/>) included teachers' resources in at least three different TSGs, specifically in TSG 18: *Analysis of uses of technology in the teaching of mathematics*, TSG 19: *Analysis of uses of technology in the learning of mathematics*, and TSG 31: *Task design and analysis*. This was the case even though the term of *resources* itself did not appear in the topic titles.

Similarly, the 2017 Congress of European Research on Mathematics Education (CERME) dedicated a thematic working group to teachers' resources for the first time (see *Curricular resources and task design in mathematics education* at <http://cerme10.org/>). Previously, some of the themes related to teachers' resources were

integrated elsewhere, mostly within the CERME activities that addressed technology in mathematics education.

During the same time period, new scientific events are being established, dedicated to research on teachers' resources broadly, and textbooks more specifically. The First International Conference on Mathematics Textbooks Research and Development (ICMT, see <http://blog.soton.ac.uk/icmtrd2014/>) was held in Southampton in 2014 (Keith et al. 2014). The Second ICMT was held in Rio de Janeiro in 2017 (<https://www.sbm.org.br/icmt2/>) and integrated a symposium on *Teacher-resource use around the world* (Remillard, Van Steenbrugge & Trouche to be published). The series of ICMT conferences will be continued in 2019 in Paderborn, Germany, and 2021 in Beijing, China. Lastly, an international conference will be held in Lyon in 2018, focusing on the theme of teachers' resources (<https://resources-2018.sciencesconf.org/>).

We would like to point out that the emergence of a field of research dedicated to teachers' resources appears as the result of a convergence of three fields, the field of research on technology in mathematics education, the field of research on textbooks (Fan 2013), and the field of curriculum resources:

- The technological evolution itself, mainly via the widespread availability of the Internet, leads the researchers in the field of technology in mathematics education to look beyond a given technological tool (a calculator, or a tablet) and beyond given software, to map out and understand a hybrid set of components and how these shape mathematical teaching and learning. These components include different hardware and software tools, but also new problem scenarios, mathematical tasks, animations, simulations, and ways of checking one's understanding, to name but a few. The word most often used for naming this hybrid set of components, imported from the Internet universe, is *resources*. An interesting insight into this word taking hold in this space is offered by tracing the evolution of vocabulary between two successive editions of the *Handbook of Mathematics Education* (2003 and 2013). Each handbook includes a chapter dedicated to technology in mathematics education. In the chapter published in the first edition of the handbook (Lagrange et al. 2003), the word “resource” appears only once, while in the chapter written 10 years later (Trouche et al. 2013), it appears 150 times.
- The evolution of textbooks through the integration of digital components, from paper textbook to e-textbook (Pepin et al. 2017), leads to an evolution of the related field of research. As a result of this evolution, each textbook is to be considered as a composed entity, and each teacher is considered as a user of a composed set of resources, exceeding a single textbook.
- The evolution of curricular resources, mainly also due to the readily available Internet, leads to a rather staggering expansion of the providers of such resources. The various institutions that traditionally provided curricular support and guidance—such as governmental bodies that oversee education—now compete in this space with various companies that might (or might not) follow the institutional prescriptions. Resources developed by experts in mathematics

education appear alongside those developed by practicing teachers, who share their own resources individually, or within frameworks established by various professional associations and online communities.

Importantly, the emergence of the new field of research is marked by the need for new conceptual and analytical tools, theoretical frameworks, and methodologies. By bringing proposals for a number of these tools, the current book witnesses the forthcoming evolutions.

The Structure of the Book

This book is organized into five parts, the final one being dedicated to concluding remarks. Parts 1 to 4 correspond to the four parts of TSG 38, which were slightly modified to reflect the focal themes of the TSG 38 discussions:

1. Trends in presentation of mathematics in textbooks and other resources;
2. Teacher interactions with curricular and other learning resources;
3. Teachers' collective work through resources;
4. Teachers' and students' interactions through resources.

The main difference between the initial structure of TSG 38 as aforementioned and the structure of the book is that we have not dedicated a section to the issues linked to digital resources. The reason is that digital resources, today, are everywhere, and they are actually present throughout the four parts of the book. The four parts are followed by a chapter of conclusions, written by the editors, and by a final discussion chapter that provides a global view on the work, by Birgit Pepin: we warmly thank her for having accepted this challenge.

Part I of the book aims to set the scene, offering a view on different kinds of mathematics teachers' resources, their variation over the time and in different cultural and institutional contexts.

In Chap. 1, Trouche, Gueudet, and Pepin propose a specific theoretical frame for analysing new phenomena arising through digitalization, in the context of France. The main phenomenon that they underline is the profusion of open educational resources (OER), which, according to the authors, "provides new opportunities for the design and use of mathematics teaching resources" (p. 3). Evidence of these opportunities is given through tracing the development of teacher associations, which increasingly design and disseminate their own teaching resources. The specific frame is the documental approach to didactics, which orients researchers to view teachers' work through the lens of their interactions with resources. This approach proposes new concepts such as teachers' resource system, which appears critical for understanding teachers' work at a time when their resources are far from being restricted to textbooks.

In Chap. 2, Qi, Zhang, and Huang, in the context of China, explore a general statement, at an international level, about the nature of textbooks moving "from an

‘authoritative course’ to a kind of ‘supporting material’ both for teachers and students” (p. 30). Drawing on the model of Nicol and Crespo (2006), distinguishing three levels of relationships between teachers and textbooks—adhering, elaborating, and creating—they analyse the work of six mathematics teachers in teaching *transformation*, a topic recently introduced in geometry textbooks in China. They provide strong evidence of the differences between teachers’ textbook usages with respect to their experience: “novice teachers mostly arrive at the adhering level while using textbooks ... As for experienced teachers’ behaviour, most of them reach the elaborating and creating level” (p. 47). These results plead for a more careful analysis of the evolution of textbook usages over time.

In Chap. 3, Fan, Mailizar, Alafaleq, and Wang present a comparative analysis of the textbook content, focusing on how geometric proof is treated in secondary school mathematics textbooks in China, Indonesia, and Saudi Arabia. By doing so, they open a window on national contexts in Asia, which are under-represented in the international research literature. This comparative analysis considers three points of view: the time and place of introducing proof in the curriculum, the distribution of the treatment of proof in the textbooks, and the type of proofs introduced to students (a classification of proofs is proposed for this last purpose). The authors document considerable differences between China on the one hand, and the two remaining countries on the other hand. The textbooks in China appear to provide a much bigger room to proof. Deepening this analysis would require, as the authors indicated, extending the analysis to the exercises proposed by the textbooks (in this chapter only the main texts of the textbooks are taken into account), and pursuing a longitudinal approach, in which the interrelated evolution of curricula and textbooks would be explored.

Part II analyses interactions between teachers and curricular and other learning resources.

In Chap. 4, Remillard proposes a theoretical and empirical contribution to understanding what she terms teacher–resource interactions. First, she proposes to define *curriculum resources* as “print or digital artefacts designed to support a program of instruction and student learning over time” (p. 71), distinguishing them from other types of resources and comprising three components: teacher’s guides, students text (books), and documents designed by teachers. Noting that teachers read and interpret this variety of components, “in order to craft instructional episodes” (p. 73), on a daily basis, Remillard builds on the notion of pedagogical design capacity (PDC) in order to better understand the complex skills involved in doing so proficiently. Empirical data from a national project in the USA are provided to illustrate the complexity of teacher–resource interactions in a case of a fourth-grade elementary teacher. In this illustrative case, the author portrays PDC as an interaction between affordances of the resource and the interpretive capacities of the teacher.

In Chap. 5, Leshota and Adler also explore the notion of mathematics teacher’s PDC. Drawing on a larger professional development study conducted with Grade 10 teachers, the authors investigate one South African teacher’s use of a prescribed textbook. They document different degrees of appropriation of the textbook and

bring to the fore the phenomena of omissions and injections of content by the teacher. Analysing these aspects and how they accompany implementation of a given lesson from a given textbook, the authors shed light on the elements of teachers' PDC, specifically the type of relationship that the teacher forges with the resource. Through their analysis, the authors propose that PDC is reflected in the quality of opportunities for mediation of mathematics that the teacher creates in using given curriculum resources.

In Chap. 6, Siedel and Stylianides analyse interviews with 36 secondary mathematics teachers in England, shedding light on teachers' resource selection processes in the context of proliferation of instructional resources. The authors document a wide range of selection processes and rationales that were shaped by resource constraints (resource accessibility and characteristics, cultural environment of instruction), mathematics topics, and teacher characteristics (personal interests and perceived student needs). The results outline new means for guiding the design of resources well fitted to teachers' needs, and for supporting the processes of selection—and appropriation—of new resources by teachers, perhaps adding a dimension to PDC that would be essential in contexts where resource selection is a teacher's responsibility.

In Chap. 7, Kynigos and Kolovou consider teachers as designers of digital educational resources for creative mathematical thinking. This contribution comes from a European project, MC2, which is further presented in Chap. 10. The authors documented how the integration of teachers into the design team, from the beginning of design process, enhanced the design of resources for benefiting and supporting creative mathematical thinking. Authors report that the design capacity of teachers appeared to be stimulated by two features of the working environment: a social feature (belonging to a community of interest that gathers a diversity of actors—teachers, researchers, and technicians) and a technological feature (benefiting from a flexible design environment and a reflective features of the tools). The collective aspect of teachers' work appears to be crucial in this analysis, and it constitutes the focus of the following part.

Part III is dedicated to teachers' collective work through resources.

In Chap. 8, van Steenbrugge, Larsson, Insulander, and Ryve propose a *collective perspective for supporting teachers' negotiation of meaning*. The issue is: to what extent is making sense of a given resource facilitated if it becomes the objective of a group of teachers? The study, grounded in a national large-scale professional development programme in Sweden, mobilized two theoretical frameworks: a social semiotic framework to analyse the meaning potential of curriculum resources, and the communities of practice framework to analyse a group of teachers' negotiation of meaning around these resources. The potential of the collective work seems to be real, developing teacher PDC in eliciting the central idea of a given resource and supporting teachers' reflective work. But the collective work seems to be more efficient for appropriating concrete ideas than abstract ones. Knowing how it is possible to combine teachers' individual learning processes and collective work around resources should require further longitudinal studies.

In Chap. 9, Wang analyses mathematics teacher expertise, curriculum resources, and collective work through windows to French and Chinese contexts. Situating her work in the documentational approach to didactics, presented in Chap. 1, she develops the notion of *documentation expertise* and suggests that it could broaden the notion of PDC, covering the whole process of teacher interacting with resources. She documents the effects of teachers' collective work on documentation expertise in two very different contexts: in China, where collective work has long been considered an inherent feature of teachers' professional activity; and in France, where teachers' collective work can be observed only in some settings.

In Chap. 10, Essonier, Kynigos, Trgalova, and Daskolia analyse the role of context in social creativity for the design of digital resources, within the frame of a European project (MC2, addressed also in Chap. 7). The authors document the productive effect of a community of interest for developing mathematical creativity. To do so, they investigate social interactions in a community of interest, composed of participants with different backgrounds, each bringing their personal experience and expertise of their respective community of practice to the shared task of instructional design. Participants' past personal experiences and expertise *create* the social context for the work of the design group. The analysis of the work of the French community of interest that was part of this project reveals the importance of the roles played by different actors (designers, as well as reviewers), the tools for supporting the design process, and the contextual factors, for collectively transforming personal experiences into designed mathematical problems.

In Chap. 11, Rocha examines teachers' experience resulting from their interactions with resources. She analyses, in the French context, the case of a mathematics teacher association, Sésamath (also addressed in Chap. 1), within which teachers collaboratively design resources. She proposes the concept of *documentational trajectory* for modelling teachers' work with resources over time and analyses the effect of collectives on these trajectories. Rocha draws on Fleck's (1981) notions of thought collective and thought style as she illustrates documentational trajectories in two analysed cases. In the first case, a new teacher was strongly involved in Sésamath, both as a designer and a user, providing an illustrative case of how a thought collective of a group designing a Sésamath textbook strongly influenced one participating designer's documentational experience and trajectory. In the second case, a teacher was involved in the Sésamath sphere as an occasional user, but her participation in several collectives positioned her as a broker of different thought styles across these collectives, making brokering a characteristic feature of her documentational experience and trajectory.

While the three previous parts focus on teachers' interactions with resources, Part IV opens a window on students, analysing the role of resources as mediating tools between teachers and students.

In Chap. 12, Ruthven examines the links between instructional activity and student interaction with digital resources. Drawing on recent studies, he analyses the way digital resources—in particular digital curriculum programmes in the case of the USA and dynamic geometry tools in the case of the UK—modify both instructional activities and student interactions through changing the task

environments in mathematics classrooms. Documenting “particular patterns of interaction between students and digital systems, and between students using such systems and their teachers” (p. 272) associated with different resources, Ruthven highlights that digital curriculum programmes, at this point, are reasonably successful in supporting student interactions with the programme system. However, they also suffer from lack of attention to facilitating peer interactions between students and student–teacher interactions. Indeed, the role of the teacher in scaffolding the interactions between students and digital resources remains crucial, making the difference between activity of producing solutions on the one hand, and that of making mathematical interpretations grounded in reflection on the solutions on the other hand.

In Chap. 13, Visnovska and Cortina analyse how resources could help teachers to plan for interactions with students' ideas. The authors focus on a particular kind of resources—instructional sequences produced in classroom design experiments—that were designed to support teachers in establishing, in their classrooms, particular types of interactions with the aim of facilitating the emergence of specific students' mathematical ideas. Building on two professional development studies conducted with teachers in the USA and in Mexico, the authors trace both specific challenges that the teachers encountered when adapting the resources to their classrooms, and supports provided within the instructional sequences, and within the co-participation structures of professional development programmes. The authors highlight the opportunities the sequence provided for (1) targeting teachers' enactment-related decisions, (2) reframing the teacher's role in instigating progress within the sequence, and (3) experiencing early success when resources were used in the teachers' classrooms.

In Chap. 14, Naftaliev examines prospective teachers' interactions with interactive diagrams. Interactive diagrams are the mathematical activities in interactive textbooks presented by software applications. The author analyses the components of such diagrams and underlines their potential for opening students' mathematical activity. Drawing from an experimental study mobilizing 25 prospective teachers (PT), she documents teachers' difficulties related to benefiting from this potential, even in situations where the teachers were aware of it. Choosing to stay within their own security zones, the PTs preferred to follow “*well-trodden paths*” (p. 311). Naftaliev uses her analysis to argue that “developing new practices and experiences in teaching-learning with interactive textbooks seems to be a necessary condition for PTs' implementation of these sources in their future teaching” (p. 312).

In Chap. 15, Kim examines teacher decisions on lesson sequence and their impact on opportunities for students to learn. When using a given resource, teachers often make adaptations to the sequence of tasks provided by the resource, for enacting a given lesson. Kim analyses two teachers' decisions in making such adaptations, and how these decisions shaped the quality of instruction and opportunities for student learning. The cases are situated within a larger study of 11 elementary teachers in the USA, who followed three different curricular programmes. Kim's analysis supports the argument that there is higher demand on

teachers who use programmes incorporating conceptual support for learners. These teachers were more likely to alter the sequence of tasks proposed by the resource, at times to the detriment of mathematical coherence of resulting learning opportunities. The author argues that teachers need to both reason *with* resources and appreciate the reasoning *in* the resources, if they were to adapt these productively to their specific classroom situations. She outlines implications of the study for curriculum designers and teacher educators.

Part V is dedicated to concluding remarks.

In Chap. 16, coordinated by Rezat, the editorial team summarizes and discusses the state of the art of research on mathematics textbooks and other resources and their use as depicted in the current volume. Based on the discussion, conclusions are drawn regarding perspectives of future research on mathematics textbooks and resources, and their use.

In Chap. 17, Birgit Pepin, as a critical friend invited by the editors, in a general commentary paper on the book, develops an argument for a complementary line of research, deserving an increased attention in mathematics education: teacher learning with educative curriculum materials.

A Broad Area of Research in Mathematics Education to be Developed

It appears clear to us that the last two decades or so have witnessed a growing interest by the international mathematics community in the study of resources, as suggested in the title of our TSG 38. Various issues related to the concept, the role, the design and the development, the use and the enactment of resources of different types such as mathematics textbooks, teaching resources, learning resources and digital resources have received increasing attention from many researchers in different parts of the world. Accordingly, new conceptualizations, new themes, and new methods have clearly emerged, providing new insights into mathematics teaching and learning, as evidenced in the chapters of this book. We hope that the book will make a meaningful contribution to the further development of this relatively new and broad area of research in mathematics education.

Finally, we wish to record our appreciation to all the contributors to the TSG 38 and the authors of this book for their contribution and all the anonymous reviewers for their generous help in reviewing all the chapters submitted for the book. We also wish to especially thank the remaining members of the organizing team, who are also co-editors of this book, for their unwavering support and commitment throughout the whole process of producing this book.

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