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# Likelihood-Free Methods for Cognitive Science

 Springer

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ISSN 2510-1889                      ISSN 2510-1897 (electronic)  
Computational Approaches to Cognition and Perception  
ISBN 978-3-319-72424-9              ISBN 978-3-319-72425-6 (eBook)  
<https://doi.org/10.1007/978-3-319-72425-6>

Library of Congress Control Number: 2017962914

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Printed on acid-free paper

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The registered company is Springer International Publishing AG  
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

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## Foreword

This book provides a concise overview of recent developments in likelihood-free inference, thereby opening a new chapter in the field of cognitive modeling. With the easy availability of computers, researchers in the field introduced a glut of mechanistic models of cognition that have no closed-form expression of the likelihood function, placing them outside of the standard statistical realm. As such, it is not generally possible to fit a mechanistic model to observed data by maximum likelihood estimation or Markov chain Monte Carlo based Bayesian methods.

Instead, as a provisional step, the best-fitting parameter values of the model of interest were estimated in the frequentist framework by a brute-force, computationally intensive implementation of least squares estimation. Obtaining the sampling distribution of a test statistic, however, is out of question. The lack of a test statistic hinders the progress of model development because without it one can neither properly assess the adequacy of the model nor compare the fitted model with other competing ones. Things are much the same in the Bayesian framework. Consequently, the technical challenge of likelihood-free inference has been a stubborn barrier to making theoretical progress in the field. This unhappy state of affairs has changed dramatically in the early 2000s with the introduction of a series of likelihood-free algorithms, such as approximate Bayesian computation (ABC), Gibbs ABC, synthetic likelihood, and probability density approximation. With the availability of these easy-to-implement methods, model evaluation and comparison for mechanistic models are now well within the reach of every modeling scientist.

The senior author of this book has been at the forefront in the development and application of likelihood-free methods to analyzing and fitting psychological models, initially applied to models of recognition memory as part of his dissertation research at the Ohio State University and subsequently expanded to other domains of modeling including episodic memory and perceptual decision making.

The book itself, written based on the authors' published articles, offers a technically comprehensive yet clearly explained account of all major likelihood-free algorithms, with a focus on Bayesian inference. Another noteworthy feature of the book is its practically oriented approach in which application examples of

the algorithms for well-known models in psychology are discussed in great detail with accompanying pseudocode and actual code as well. The work such as this will make a transformative impact on the advancement and practice of computational modeling of cognition. This book is a first positive step in that direction.

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September 2017

Jay Myung

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## Acknowledgments

This work was supported by an NSF grant SES-1424481 awarded to Van Zandt, and an NRSA grant 1F32GM103288-01 awarded to Turner.

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