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Martin Auer

# Hands-On Value-at-Risk and Expected Shortfall

A Practical Primer

 Springer

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## Endorsements

A very useful guide to the theoretical and practical aspects of implementing and operating a risk-monitoring system for a mid-size financial institution. It sets a common body of knowledge to facilitate communication between risk managers, computer and investment specialists by bridging their diverse backgrounds.

**Giovanni Barone-Adesi**

Professor, Università della Svizzera italiana  
Lugano, Switzerland

This unassuming and insightful book starts from the basics and plainly brings the reader up to speed on both theory and implementation.

**Shane Hegarty**

Director Trade Floor Risk Management  
Scotiabank

Visit the book's website at [www.value-at-risk.com](http://www.value-at-risk.com).

*For  
Nahuel,  
Mirjam,  
Yuna,  
Yannik,  
and Julian*

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## Preface

This book sets out to describe the simplest market risk model that is still practical. It outlines the model's underlying math, daily operation, and implementation, while stripping away as much statistical overhead as deemed fit. It does not advance some novel methods but attempts to pick and present those modeling approaches and methods that might conceivably do the job. We set up and operate a highly similar model in a mid-sized Austrian bank; it performed very well overall, right through some turbulent years in financial markets. Of the calls we made, we got a few right. Some we didn't at first, but we fixed them and hopefully learned from them. They all came to shape this account.

I wrote this book for the fun of it. But why should you read it? If you are a recent graduate on your first day in office or a time-starved manager ready to brush up on your market risk fundamentals, you hopefully get an easy-to-digest introduction to basic risk measures and their properties. If you are a programmer, you might learn about the mathematical underpinnings of your code, making your exchanges with the risk department just a bit smoother. If you are a quant, maybe you can gauge the types of support statistics most useful to daily risk operation. If you are a team leader allocating money and time, you possibly find ways to steer through the technical jargon and rein in the understandable compulsion of your team to use the latest tech and the fanciest math. If you operate the model on a daily basis, some of the analytical support methods given here might help you understand, explain, and defend the numbers. If you're anyone, really, in this motley crew, I hope this book will ease your communication with colleagues, clients, and controllers.

After my studies, I quickly figured out that college is far more fun than work, so I applied to several programs at New York universities. By chance, I got admitted to a financial mathematics one, where in my very first lecture I discovered this “bond” thing, an alien concept in my cash-strapped student life. That other people may likewise get exposed to market risk topics after varying journeys set up this book's angle. This book at times will state the obvious and thus often unmentioned—even apparently trivial calls are easier to make if their trail is known to be trodden. It will appear sloppy at times, as it proposes heuristics rooted in the nature of imperfect real-world data. While it tries to offer a consistent notation, it will gladly gloss over many technical details. Several opinions in it you will find judgmental; indices missing in a sacrifice to readability; and shortcuts taken to save on paper and

rumination. (And those are just the infractions I committed on purpose.) The views given here are mine and do not represent those of my current or former employers.

I meant this book, in the end, to be about simplicity and about communication between team members of wildly differing backgrounds. I hope reading it is worth a bit of your while.

Vienna, Austria  
2018

Martin Auer



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