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S.M.A.R.T. Circle Minicourses

 Springer

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This book is dedicated to my comrades in the outreach front line:

Maria Falk de Losada

Nikolay Konstantinov

József Pelikán

Zonghu Qiu

Wen-Hsien Sun

Peter J. Taylor

Paul Vaderlind

Preface

A Brief History of the S.M.A.R.T.Circle

A most beneficial side effect of the collapse of the former Soviet Union in 1992 was the migration of the Mathematical Circles across the Atlantic to the United States. Mathematical Circles, originated in Hungary during the nineteenth century, are a glorious tradition in Eastern Europe. They are organizations which discover and nurture young mathematical talents through meaningful extra-curricular activities.

The process took a few years, leading to the formation in 1998 of the Berkeley Mathematical Circle. With the support of the Mathematical Sciences Research Institute, the movement has caught fire in the United States, culminating in the formation of a Special Interest Group in the Mathematical Association of America under the leadership of Tatiana Shubin of San Jose State University.

Unbeknown to this community, a Mathematical Circle had existed in North America almost two decades earlier. The ultimate inspiration was still of Soviet origin, but the migration took place across the Pacific, via the People's Republic of China in the form of their Youth Palaces. This was the S.M.A.R.T. Circle in Edmonton, Canada, founded in 1981. The acronym stood for Saturday Mathematical Activities, Recreations & Tutorials.

I was born in China during the over-time sudden-death period of the Second World War, but moved to Hong Kong at age six. Thus I had never attended any session of any Youth Palace. However, I followed reports of their activities, and this fueled my interest in mathematics. The first mathematics book I had was a Chinese translation of Boris Kordemski's *Moscow Puzzles*, which was on their recommended reading list. An English version is now an inexpensive Dover paperback. Later, I acquired Chinese translations of several wonderful books by Yakov Perelman. Dover has published his *Figures for Fun* in English.

I came to Canada at age twenty, and eventually got a tenure-track position at the University of Alberta in 1980. That fall, I was invited to a general meeting of the Edmonton Chapter of the Association for Bright Children. My comment was that their activities seemed heavily biased towards the Fine Arts. Having put my foot in my mouth, I was obliged to take some concrete action. The next spring, the S.M.A.R.T. Circle was born.

During the first year, the members ranged from Grade 3 to Grade 6, because of the clientele of the A.B.C. However, to do meaningful mathematical activities, I preferred the children to be a bit more mature. So the Grade level rose by one each year, until in 1985, the members ranged from Grade 7 to Grade 10. Many of them stayed throughout this period.

As we moved away from the normal age of the clientele of the A.B.C., the Circle practically became an independent operation. This also became necessary because in 1983, we received a grant of \$1,500 from the University of Alberta, arranged by Vice-President Academic **Amy Zelmer**. With the money, I built up a Circle Library containing mathematical books, games and puzzles. This was the only funding the Circle had received in its thirty-two year history.

We met on the University of Alberta campus from 2:00 pm to 3:00 pm every Saturday in October, November, February and March. A second classroom adjacent to the meeting room was open from 1:30 pm to 3:30 pm as the Circle Library. **Adrian Ashley**, a former Circle member, was hired at \$5 an hour to look after it. There was a comedy of error in that for a while, his salary came out of the Student Union cafeteria account! They soon put a stop to that, but never bothered to claim readjustment.

Because of the members' tender ages, most came with their parents, and some parents stayed in Circle Library during the session. Members also had half an hour before and half an hour after the session to browse through. Sometimes, some younger members' attention span wandered during the session, and they would drift to the Circle Library for a few minutes.

In 1986, the three-year period of the grant ran out. As I closed the account, I turned the Circle Library over to the Faculty of Education. Then I started building a replacement out of my own pocket. Meanwhile, the A.B.C. had acquired new headquarters in the form of a house, where the basement was set up as a classroom. The Circle was invited to move its operation there. As a result, I restarted the session for A.B.C. members from Grade 3 to Grade 6 again. This went from 1:00 pm to 2:00 pm while the existing session for the older children ran from 2:30 pm to 3:30 pm. We had quite a few sibling pairs. Sometimes, one was in class in the basement while the other waited upstairs and played with mathematical games and puzzles from the new Circle Library. Sometimes, they sat in the same session despite any disparity in age.

In 1991, this arrangement came to an end, and the Circle moved back to the university campus. Only the Grade 7 to Grade 10 session survived the move. The meeting time was once again from 2:00 pm to 3:00 pm. A section at the back of the classroom was reserved for the Circle Library.

In 1996, there was a reverse migration of the Circle movement back across the Pacific, to Taiwan. My friend Wen-Hsien Sun of Taipei started the Chiu Chang Mathematical Circle, initially based on my model and using much of the material I had accumulated over a decade and a half. Both Circles closed in 2012, though mine was reincarnated as the J.A.M.E.S. Circle, standing for Junior Alberta Mathematics for Eager Students. It is run by my former student **Ryan Morrill**.

The activities of the S.M.A.R.T. Circle may be loosely classified into the following overlapping categories:

1. **Mathematical Conversations;**
2. **Mathematical Competitions;**
3. **Mathematical Congregations;**
4. **Mathematical Celebrations.**

The first two activities are the heart and soul of our Circle right from the beginning. The last two did not emerge until the second half of our Circle's thirty-two year history. For a description of these activities, see the companion volume *The S.M.A.R.T. Circle — Overview*.

The Mathematics Conversations are the heart and soul of the Circle. There is a Fall Session and a Winter Session each academic year. The Fall Session runs in October and November while the Winter Session runs in February and March. We meet every Saturday during those months from two to three in the afternoon at the University of Alberta. Each session consists of a minicourse plus a number of investigation topics. The latter lead to projects by Circle members, either independently or in small groups.

The material in this book is based on the best received minicourses offered over the thirty-two year history of the Circle, both in the Mathematical Conversation sessions and in the summer camps under Mathematical Congregations.

Andy Liu,
Edmonton, Alberta, 2017.

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