

Solid Mechanics and Its Applications

Volume 250

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Contact Mechanics

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Preface

For many years, contact mechanics was more or less synonymous with Hertzian contact where the contacting bodies are elastic with quadratic profiles, and most of the applications were to traditional engineering components such as rolling bearings, cams and gears. However, more recent applications cover an extraordinarily diverse range, including natural and artificial hip joints, the slip of tectonic plates during earthquakes, the adhesion of gecko feet to a wall when climbing, the interpretation of atomic force microscope (AFM) results and many others. Indeed in almost all systems comprising more than a single object, loads are transmitted between the components by contact, and the nature of this interaction is often critical in determining the overall system behaviour.

For ‘macroscopic’ systems, the contact interaction can usually be simplified by defining a dichotomy between the states of contact and separation. Bodies in contact can transmit loads and conduct heat and electricity, whereas these processes are either impossible or much reduced if the bodies are separated by even a small gap. In idealized models, this distinction typically translates into a problem governed by inequalities and the resulting strong nonlinearity is a rich source of interesting and complex mathematical phenomena. Additional inequalities are introduced through the transition from stick to slip in problems involving friction.

However, recent applications increasingly involve very small length scales, where the distinction between contact and separation is blurred, and we must also recognize the possibility of tensile [adhesive] tractions between the bodies, for example, due to van der Waals forces. In such cases, the inevitable roughness of the contacting surfaces plays an important rôle. In biological applications, the materials are likely to experience large strains and the characterization of the material constitutive law is a challenge. Also, at extremely small scales, we must recognize that continuum formulations will become inappropriate.

These considerations imply that contact problems are likely to be encountered by scientists who are not primarily interested in contact mechanics *per se*, and that the resulting problems can be quite complex, usually necessitating numerical solution methods. Fortunately, modern finite element packages are very user-friendly and increasingly contain modules describing a range of multiphysics interactions

between surfaces. However, the user will often encounter unexpected [and sometimes unexpectedly simple] predictions, and this always raises the question as to whether an idealized analytical treatment may be sufficient to capture and indeed ‘explain’ the qualitative behaviour of the system, whilst providing a greater level of generality and hence predictive power. Analytical models also have the benefit of identifying the principal determinants of the behaviour of the system [particularly when supplemented with finite element predictions for the same system], and hence provide guidance as to which features of the underlying physics or geometry require especially careful characterization or measurement.

These considerations have influenced the presentation of this book. Although some relatively complicated mathematical treatments are addressed, there is an emphasis on qualitative physical behaviour, and on situations where a simpler approach gives useful results. Also, I have included problems at the end of each chapter, principally as an indication to the reader of how the methods and concepts discussed can be applied in different systems, though these problems are also suitable as assignments in a course on contact mechanics.

The field of contact mechanics has expanded considerably since K. L. Johnson published his classical monograph in 1985, and arguably it would be impossible to achieve the same level of completeness in a single book today. I therefore make no apology for focussing on topics with which I have had first-hand experience, and this [rather than any misplaced sense of self-importance] explains the high proportion of citations to my own work and to that of my graduate students and collaborators. In particular, most of the book relates to linear elastic materials and there is relatively little discussion of interior stress fields, even though these can be of importance for failure analysis, particularly with reference to surface durability. Other significant omissions include numerical methods, lubrication, plasticity and viscoelasticity.

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