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Tiziano Squartini · Diego Garlaschelli

Maximum-Entropy Networks

Pattern Detection, Network Reconstruction
and Graph Combinatorics

 Springer

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*To Angela and Costanza
our welcoming horizons*

*A Leonardo e Valeria
due Stelle luminose
che camminano sulla Terra*

Preface

*«... i' vegno per menarvi all'altra riva
ne le tenebre eterne, in caldo e 'n gelo.
E tu che se' costì, anima viva,
pàrtiti da cotesti che son morti.
Ma poi che vide ch'io non mi partiva,
disse: «Per altra via, per altri porti
verrai a piaggia, non qui, per passare:
più lieve legno convien che ti porti».
E 'l duca lui: «Caròn, non ti crucciare:
vuolsi così colà dove si puote
ciò che si vuole, e più non dimandare».*

—Dante Alighieri, *Divina Commedia*

This book is intended as a complete, self-consistent introduction to a general methodology to study complex networks. This methodology combines concepts of information theory, statistical physics, and graph theory and provides a way to build *maximum-entropy models of networks*. These models have a rigorous theoretical origin and a range of practical applications. In this book, we emphasize the applications to pattern detection, network reconstruction, and graph combinatorics.

Most of the results in this book have been developed by ourselves and our direct collaborators. Our approach has been rigorously investigated both analytically and numerically and used in several applications, ranging from physics to economics and biology. Given the robustness of the method, its wide range of applicability, and the growing interest it has been attracting from our colleagues, we felt the need of writing a compact text unifying our results fragmented across many publications.

Pattern detection, network reconstruction, and graph combinatorics are by themselves three distinct and active fields of research. Several specialized and generally unrelated techniques have been proposed within each of these fields. Our

aim in this present book is *not* that of discussing the existing field-specific approaches, but that of emphasizing the *connections* between these seemingly independent problems.

Moving from common, general, first principles, we present a unified methodology which provides various explicit solutions to the aforementioned problems. Thus, while providing the references necessary to contextualize all the results discussed here, this book is deliberately not in a review-like structure. Rather, it focuses on an original, unifying approach to these three selected domains of network theory.

This book is intended for a broad audience, ranging from Ph.D. students in physics and mathematics looking for an unconventional introduction to network theory, to researchers in other disciplines (e.g., economics, social sciences, biology) interested in the application of network analysis to their topic(s) of interest. No prior knowledge of network theory or any of the specific topics discussed here is required.

Lucca, Italy
Leiden, The Netherlands
May 2017

Tiziano Squartini
Diego Garlaschelli

Acknowledgements

*Frodo: But I am going to Mordor.
Sam: I know that well enough, Mr. Frodo.
Of course you are. And I'm coming with you.*

—John Ronald Reuel Tolkien, *The Fellowship of the Ring*

We are indebted to a number of friends and colleagues who have collaborated with us in the development and application of the methodology presented in this book. Large parts of these results would have not been obtained without their help and dedication.

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