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Vivishek Sudhir

Quantum Limits on Measurement and Control of a Mechanical Oscillator

Doctoral Thesis accepted by
École Polytechnique Fédérale de Lausanne, Switzerland

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The hardest thing of all to see is what is really there.

J.A. Baker, *The Peregrine*

*To my father, who by patiently answering all
my questions, became my first science teacher,
And to my mother, who taught me everything else*

Supervisor's Foreword

The study of radiation pressure coupling between light and motion in optical interferometers has a long history, one that predates its study in atomic systems. The seminal work of Braginsky in the 1960s predicted that the radiation pressure of light in an optical interferometer causes two disturbances that limit the ability to measure optical displacements: a classical—dynamic back-action—that causes parametric instability of the mirror, and a quantum mechanical limit imposed by the quantum fluctuations of the radiation pressure force. While originally formulated within the context of interferometric gravitational wave detection, both classical dynamical back-action and its quantum mechanical counterpart have become observable in experiments using cavity optomechanical systems that utilize high-Q and small mass mechanical oscillators coupled to intense optical fields stored in cavities. While dynamical back-action physics, exemplified by laser sideband cooling of massive mechanical oscillators, has been accessed by a wide range of nano- and micron-scale optomechanical systems, observing the limits imposed by the quantum nature of light has been far more challenging. The latter is compounded by the fact that the quantum fluctuations of the radiation pressure force are faint, and further, they can be easily masked by classical noises.

The present thesis from Vivishek Sudhir constitutes the first experiments carried out in our laboratory at EPFL that observes and studies the quantum nature of the radiation pressure interaction. This is achieved via advances in the ability to perform sensitive measurements and to operate novel nano-optomechanical systems at low cryogenic temperatures. These advances enable a series of experiments that demonstrate the origin and physical effects that result from radiation pressure quantum noise. A newly developed optomechanical system consisting of a high-Q small mass nanostring dispersively coupled to the evanescent field of an optical whispering gallery mode resonator enabled to achieve vacuum cooperativities near unity, translating into an ability to measure the motion of the nanostring with an imprecision at the standard quantum limit (SQL) with less than a single intracavity photon on average. In fact, by engineering the nanostring to oscillate at frequencies untouched by other measurement noises, it was possible to measure its motion with an imprecision 40dB below that at the SQL. With such record measurement

sensitivity, the oscillator could be feedback cooled to an occupation of only 5 quanta (corresponding to 16% ground-state occupation). Although feedback cooling had been proposed as a method to reduce the thermal motion of mechanical oscillators since decades, its utility for reducing thermal noise to a level comparable to the zero-point motion of the oscillator had to wait till this thesis.

This experiment sets the stage for the exploration that is carried out in the rest of the thesis—a dissection of linear quantum measurements. The ability to cool the mechanical oscillator to such low occupancy implies that the corresponding quantum back-action, resulting from the radiation pressure quantum fluctuations, has been canceled. Therefore, feedback cooling to occupancies below that due to back-action should more accurately be viewed as an example of quantum feedback. In a subsequent series of experiments, this thesis goes on to precisely explore the different manifestations of this suppressed quantum back-action. Quantum back-action is shown to give rise to two effects: pondermotive squeezing of light and motional sideband asymmetry. A particularly interesting aspect of the work is that both effects are observed in one and the same experiment for the first time, depending on how one analyzes the post-measurement optical field: in case of homodyne detection, one observes that the strong measurement causes squeezing of the light, while for the case of heterodyne detection (combined with feedback cooling) the experiments reveal an asymmetry in the sidebands scattered by the mechanical oscillator. The work therefore shows that the two effects have precisely the same origin: the generation of correlations between the amplitude and the phase fluctuations of the measurement laser. Measuring such quantum correlations is a daunting task, and the experimenter has to exercise extreme care to rule out classical effects. Both manifestations of quantum correlations can be mimicked by classical noises—the difference lying only in their calibration. The present thesis achieves this in a particularly elegant way: operation with large vacuum cooperativity in the Doppler regime combined with feedback cooling eliminates a large number of classical measurement noises; in particular, and counterintuitively, the experiment is not sensitive to classical phase noise of the measurement lasers. Moreover, significant sources of systematic error are eliminated by being able to observe sideband asymmetry by only varying the electronic gain of the feedback path, and not the laser power or detuning. These advantages together allow to demonstrate a quantum mechanical sideband asymmetry at the level of 10% in agreement with the occupation of the oscillator. Together, optical squeezing and sideband asymmetry show the quantum correlations induced by an optical field by a “macroscopic” mechanical oscillator. The culmination of the thesis is to probe, using sideband asymmetry, the regime where feedback of detected quantum noise leads to noise squashing. In this regime, the sideband asymmetry disappears. On the one hand, it corroborates the trustworthiness of the calibration of classical measurement noises. On the other hand, it highlights a basic limitation of quantum feedback: although it can cancel back-action caused by quantum noise, it cannot overcome detection quantum noise.

Overall, the present thesis represents the first experiments at the laboratory at EPFL where long-predicted quantum effects of radiation pressure were probed

and analyzed. This thesis provides the first glimpse into the quantum nature of mechanical oscillators, and a step toward quantum control of macroscopic mechanical systems, after that of atoms, ions, and superconducting circuits in recent decades.

Lausanne, Switzerland
August 2017

Prof. Tobias J. Kippenberg

Abstract

The precision measurement of position has a long-standing tradition in physics. Cavendish's verification of the universal law of gravitation using a torsion pendulum, Perrin's confirmation of the atomic hypothesis via the precise measurement of the Brownian motion, and the verification of the mechanical effect of electromagnetic radiation all belong to this classical heritage. Quantum mechanics posits that the measurement of position results in an uncertain momentum; an idea developed to full maturity within the context of interferometric searches for gravitational waves. Over the past decade, standing at the confluence of quantum optics and nanomechanics, cavity optomechanics has emerged as a powerful platform to study the quantum limits of position measurements.

The subject of this thesis is the precision measurement of the position of a nanomechanical oscillator, the fundamental limits of such measurements, and its relevance to measurement-based feedback control. The nanomechanical oscillator is coupled to light confined in an optical micro-cavity via radiation pressure. The fluctuations in the position of the oscillator are transduced onto the phase of the light, while quantum fluctuations in the amplitude of the light lead to a disturbance in the momentum of the oscillator. We perform an interferometric position measurement with a sensitivity, that is, 10^5 times below what is required to resolve the zero-point motion of the oscillator, constituting the most precise measurement of thermal motion yet. The resulting disturbance—measurement back-action—is observed to be commensurate with the uncertainty principle, leading to a 10% contribution to the total motion of the oscillator.

The continuous record of the measurement (performed in a 4K cryogenic environment) furnishes the ability to resolve the zero-point motion of the oscillator within its decoherence rate—the necessary condition for measurement-based feedback control of the state of the oscillator. Using the measurement record as error signal, the oscillator is cooled toward its ground state, resulting in a factor 10^4 suppression of its total (thermal and back-action) motion, to a final occupation of 5 phonons on average.

Measurements generally proceed by establishing correlations between the system being measured and the measuring device. For the class of quantum measurements employed here—continuous linear measurements—these correlations arise due to measurement back-action. These back-action-induced correlations appear as correlations between the degrees of freedom of the measuring device. For interferometric position measurements, quantum correlations are established between the phase and amplitude of the light. In a homodyne measurement, they lead to optical squeezing, while in a heterodyne measurement, they appear as an asymmetry in the sidebands carrying information about the oscillator position. Feedback is used to enhance sideband asymmetry, a first proof-of-principle demonstration of the ability to control quantum correlations using feedback. In the regime where amplified vacuum noise dominates the feedback signal, the disappearance of sideband asymmetry visualizes a fundamental limit of linear feedback control. Using a homodyne detector, we also characterize these quantum correlations manifested as optical squeezing at the 1% level.

Keywords Quantum Measurement · Cavity Optomechanics · Quantum Feedback · Quantum Correlations

Acknowledgements

It seems to me I am trying to tell you a dream—making a vain attempt, because no relation of a dream can convey the dream-sensation, that commingling of absurdity, surprise, and bewilderment in a tremor of struggling revolt, that notion of being captured by the incredible which is of the very essence of dreams...

Joseph Conrad, Heart of Darkness

Experimental physics demands a certain degree of manual dexterity, the patience to tolerate the mundane, and an inexhaustible supply of optimism. Tobias hired me to work on an immensely sophisticated experiment despite any proof of my possessing these qualities; I am indebted to him for the belief he has placed in me. I would also like to acknowledge his dedication to fostering an environment where the pursuit of science is unencumbered by other worldly concerns. Stefan, Ewold, and Samuel bore the brunt of a theorist trying to perform delicate experiments; the stern, yet encouraging, stewardship of this trio ensured that I enjoyed the culture shock. Dal Wilson was more than a colleague and a post-doc; his pragmatic approach to physics provided the ideal counterpoint in our attempts to forge a deeper understanding of things ranging from vibration isolation to the subtleties of quantum mechanics. Without the patient efforts of Amir, Ryan, and Hendrik in designing, fabricating, and testing of devices, none of the results reported here would have been possible. I hope I have been able to bequeath a better vision of the future of our experiment to Sergey. Conversations with Alexey, Daniel, and Nathan have enabled me to vicariously learn how to measure microwave “photons”. John Jost once taught me how to decorate a Christmas tree; since then he has imparted some wisdom on frequency metrology, and some on atomic physics. Together with Dal, Victor and Caroline have probably spent the most time with me outside the lab; it was fun to exercise an air of social normalcy with this bunch. Christophe has been an amazing sparring partner on every subject capable of being brought under scientific scrutiny.

The personal space required for the pursuit of science comes through the sacrifice of many people. My family back home in India—parents, sister, and grandparents—have had to patiently wait for the brief interludes when I go home. No words can do justice to this and the very many other pleasures they have surrendered over the years for my sake. Before physics attracted me, I was charmed by someone else; I am lucky to have married her. Longtime friends, mentors—Ajith and Subeesh—have played pivotal roles in my being able to engage in physics; I hope to repay this enormous debt, some day.

It was a privilege to have learnt from a few great teachers during my formative years. Their vision of an indelible unity in nature continues to motivate my study of physics.

Contents

1 Prologue	1
1.1 Precise Position Measurements	4
1.2 Outline of this Thesis	8
1.2.1 Organization of Thesis	9
References	9
2 Quantum Fluctuations in Linear Systems	13
2.1 Kinematics of Fluctuations in Quantum Mechanics	14
2.1.1 Operational Description of Fluctuations in Time	16
2.1.2 Spectral Densities and Uncertainty Relations	18
2.2 Dynamics Due to a Thermal Environment	23
2.2.1 Effect of Fluctuations from a Thermal Environment	26
2.3 Dynamics Due to a Meter	29
2.3.1 Effect of Fluctuations from a Meter	30
References	32
3 Phonons and Photons	35
3.1 Phonons: Quantised Linear Elastodynamics	35
3.1.1 Classical Description of Navier-Euler-Bernoulli Elastic Field	37
3.1.2 Quantised Modes of the Elastic Field	41
3.1.3 Mechanical Oscillator in Thermal Equilibrium	43
3.2 Photons: Description and Detection	46
3.2.1 Quadrature, Number, and Phase Operators	49
3.2.2 Quantum and Classical Fluctuations in the Optical Field	50
3.2.3 Detection of Optical Fluctuations	54
3.2.4 From Propagating Modes to Standing Waves: Optical Cavity Coupled to a Waveguide	69
References	79

4 Photon-Phonon Coupling: Cavity Optomechanics	83
4.1 Perturbing an Optical Cavity	83
4.2 Effective Description: Single-Mode Cavity Optomechanics	87
4.2.1 Steady-State Shifts	89
4.2.2 Dynamical Back-Action	90
4.3 Continuous Linear Measurement Using Cavity Optomechanics	94
References	100
5 Experimental Platform: Cryogenic Near-Field Cavity Optomechanics	103
5.1 Stressed Nanostring Coupled to an Optical Microcavity	104
5.1.1 Near-Field Coupling	106
5.1.2 Mechanical Properties of Stressed Radio-Frequency Beams	108
5.2 Measurement and Calibration of Thermomechanical Motion	111
5.3 Cryogenic Operation	116
5.3.1 Nature of Elastic Force: Radiation Pressure Versus Thermoelasticity	121
5.4 Experimental Schematic	123
References	125
6 Observation and Feedback-Suppression of Measurement Back-Action	127
6.1 Quantum-Noise-Limited Position Measurement	128
6.1.1 Measurement Imprecision and Back-Action in a Split-Mode Cavity	129
6.1.2 Measurement Imprecision	135
6.1.3 Measurement Back-Action	142
6.2 Feedback Suppression of Back-Action	145
6.2.1 Synthesis of a Linear Quadratic Gaussian Controller	147
6.2.2 Feedback by Cold Damping	153
6.2.3 Implementation of Feedback	156
6.2.4 Feedback Cooling to Near the Ground State	160
6.3 Conclusion	161
References	162
7 Observation of Quantum Correlations Using Feedback	165
7.1 Quantum Correlations Due to Light-Motion Interaction	166
7.1.1 Manifestation as Ponderomotive Squeezing	167
7.1.2 Manifestation as Sideband Asymmetry	169
7.2 Observation of Quantum Correlations	170
7.2.1 Observation of Ponderomotive Squeezing	170
7.2.2 Observation of Sideband Asymmetry Using Feedback	175

- 7.3 Conclusion 188
- References 189
- 8 Epilogue 191**
 - 8.1 Quantum Correlations for Metrology and Control 192
 - References 195
- Appendix A: Uncertainty Inequalities 197**
- Appendix B: Miscellanea on Elastodynamics 201**
- Appendix C: Response of an Imbalanced Interferometer 211**