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Melvyn B. Nathanson  
Editor

# Combinatorial and Additive Number Theory II

CANT, New York, NY, USA, 2015 and 2016

 Springer

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# Preface

The CUNY Graduate Center Workshops on Combinatorial and Additive Number Theory (CANT) have been organized every year, beginning in 2003, by the New York Number Theory Seminar. The seminar was started in 1981 by David and Gregory Chudnovsky, Harvey Cohn, and Mel Nathanson, and for 36 years has been meeting every Thursday afternoon during the academic year, and also in the summer.

The four-day CANT conferences are held in May at the CUNY Graduate Center in Manhattan, usually from Tuesday to Friday of the week immediately preceding Memorial Day. They have become a fixed point in the number theory calendar.

This collection derives from talks at the CANT 2015 and CANT 2016 workshops. There are 20 papers on important topics in number theory and related parts of mathematics. These topics include sumsets, partitions, convex polytopes and discrete geometry, Ramsey theory, primality testing, and cryptography.

I am grateful to Springer and its mathematics editor, Marc Strauss, for publishing the proceedings of these meetings. A previous volume is [1].

Bronx, NY, USA

Melvyn B. Nathanson

## Reference

1. M.B. Nathanson, editor, Combinatorial and additive number theory–CANT 2011 and 2012. Springer Proc. Math. Stat. vol. 101, Springer, New York, 2014

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