# Advanced Structured Materials

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# Asymptotical Mechanics of Composites

## Modelling Composites without FEM

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# Preface

*Complex models are rarely useful (except for those writing their dissertation)*
—V.I. Arnold.

A foreword section is usually written to convince readers that they must buy the book. However, in our case, it seems to be not easy task. Any researcher, who gives a keyword "composite material" for Internet search obtains numerous books and papers devoted to this subject. Hence, a question arises, why our book should be still interesting to the readers? In what follows, we briefly describe the characteristic and novel features of our monograph.

Our monograph is mainly spanned on three basic notions: "composite material", "internal structure" and "asymptotic methods". Composites strongly influence our lives, and hence, we skip here description of their popularity and importance. On the other hand, a natural question appears: What does it means a composite material? We are based here on our intuition and employ the following working term: composites are materials composed of a few components with different physical-mechanical characteristics. Owing to the optimal choice of the composite components, its volume ratio and geometrical forms, it is possible to fabricate new materials exhibiting valuable properties: high strength and stiffness accompanied by small volume weight, improved heat- and electroconducting characteristics, resistance to action of the aggressive matters. Theory of composite covers a wide of range of materials; such a range is of obvious importance to almost all types of engineering disciplines, including aeronautics, metallurgy and civil engineering to mention just a few of them.

The notion "internal structure" seems to be also intuitively clear. Dealing with a composite material, one may study its global characteristics (for instance, elastic modulus of sufficiently large volume of a composite) as well as the local distributions of the material additives. The latter are defined by real (internal) composite structure.

The last term to be discussed reads “asymptotic methods”. Since this approach is mainly employed in this book making it different from the other existing works on the market, we stop on this notation a little bit longer.

The term “asymptotic” (in Greek it means “not coinciding”) is linked with a clear geometric interpretation of a line, which is approached by a curve tending to it in infinite way but never reached it. The idea of asymptotic approximation stands for one of the important and deep mathematical notions and has common points with physics, mechanics and engineering. The reason is that any physical theory formulated in a general/sophisticated manner is complex and difficult for understanding from the mathematical point of view. Therefore, at the origins of the theory as well as in its further developments a crucial role is played by some limiting/particular cases allowing to find analytical solutions. Having in hand those limiting cases, one may usually decrease a number of the studied equations, decrease their order, find a transition from a discrete to continuous matter or from non-homogeneous matter to homogeneous one, etc. Behind all those idealizations and in spite of their richness and differences, typically a high-order symmetry associated with the derived mathematical model in the limiting situation usually occurs. Now, an asymptotic approach to complex problems relies on treatment of the input system (not ideally symmetric) as that being closed to a certain symmetric system. The principal advantage of the asymptotic approach is based on estimation of the correcting terms to the known limiting case in essentially easier way than the direct investigation of the input system. It seems on the first glance that the possibilities of this approach are bounded by narrow intervals of changes of the system parameters. However, experience in investigation of numerous physical problems shows that while changing essentially the system parameters causing a shift of the problem from the limiting symmetric case, usually there exists another limiting system though usually with less exhibited symmetry but still allowing for construction of a solution. This enables to describe the system behaviour in the whole interval of change of the parameters being based only on a few limiting cases.

The recently observed increase of interest towards the asymptotic methods seems to be opposite to the observed development of computational mathematics. The reason is that the asymptotic methods always develop our intuition, and hence, they play an important role in the formulation of thinking of the today's researcher or engineer. Even in the cases, where we are aimed only at getting numerical results, the initial asymptotic analysis allows to suggest a choice of most suitable computational method as well as to logically organize the obtained numerical material. Besides, this kind of analysis is particularly effective regarding those values of parameters, where direct computer numerical calculations meet serious difficulties. The latter positive aspect of the asymptotic methods has been outlined by D. Crighton [1]: “Design of computational or experimental schemes without the guidance of asymptotic information is wasteful at best, dangerous at worst, because of possible failure to identify crucial (stiff) features of the process and their localization in coordinate and parameter space. Moreover, all experience suggests that asymptotic solutions are useful numerically far beyond their nominal range of validity, and can often be used directly, at least at a preliminary product design stage, for example, saving the need for accurate computation until the final design stage where many variables have been restricted to narrow ranges".

Effectiveness of the asymptotic methods, like the homogenization approach, employed in theory of composites has been recognized for many years. However, they are rather used on the defined stage of computation, and usually, they are combined with the numerical approaches. We are aimed at the following target. Beginning working with asymptotic methods it makes sense to reduce a solution to the most simplest computational formula. However, this general idea requires searching for additional small/perturbation parameters, and there is a need to use various formulas for summation and interpolation, as well as to extend the space of applicability of known asymptotic approaches, etc. The so far mentioned idea served for us as a general base while writing this book.

Let us refer to the potential market of readers. Since we are aimed at reaching simple and simultaneously accurate formulas, they can be directly employed by engineers and designers. It means that they are not forced to study the mathematical formalisms yielding the derived formulas.

We hope that researchers recognized as the mechanics analysts, being mainly interested in investigation of composite materials, can be attracted by the presented asymptotic approaches. We hope that the book will be useful also to physicists and mathematicians who are interested in composites. It seems that although there is an exchange of ideas among mechanics, physics and mathematics, there exist still examples of "discovering" known methods in a given branch of sciences being already known in other one.

It is assumed that the reader has knowledge of basic calculus as well as the elementary properties of ODEs and PDEs, strength of materials and theory of elasticity. Finally, we tried to write the book being self-sufficient and user-friendly, putting advance knowledge into tutorial. Equipped with the necessary tools from tutorial, the readers can begin working with the objects covered by the theme of theory of composites.

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We are aware of the fact that the book may contain controversial statements, too personal or one-sided arguments, inaccuracies and typographical errors. Any of remarks, comments and criticisms regarding the book are appreciated.

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