

# Statistical Physics for Electrical Engineering

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# Preface

This short book is based on lecture notes of a course on statistical physics and thermodynamics, which is oriented, to a certain extent, toward electrical engineering students. The course has been taught in the Electrical Engineering department of the Technion (Haifa, Israel) ever since year 2013. The main body of the book is devoted to statistical physics, whereas much less emphasis is given to the thermodynamics part. In particular, the idea is to let the important results of thermodynamics (most notably, the laws of thermodynamics) to be obtained as conclusions from the derivations in statistical physics.

Beyond the variety of central topics in statistical physics that are important to the general scientific education of the electrical engineering student, special emphasis is devoted to subjects that are vital to the engineering education concretely. These include, first of all, quantum statistics, like the Fermi–Dirac distribution, as well as diffusion processes, which are both fundamental for deep understanding of semiconductor devices. Another important issue for the electrical engineering student is to understand mechanisms of noise generation and stochastic dynamics in physical systems, most notably, in electric circuitry. Accordingly, the fluctuation–dissipation theorem of statistical mechanics, which is the theoretical basis for understanding thermal noise processes in systems, is presented from a signals-and-systems point of view, in a way that would hopefully be understandable and useful for an engineering student, and well connected to some other important courses learned by students of electrical engineering, like courses on random processes. The quantum regime, in this context, is important too and hence provided as well. Finally, we touch very briefly upon some relationships between statistical mechanics and information theory, which is the theoretical basis for communications engineering, and demonstrate how statistical–mechanical approach can be useful for the study of information–theoretic problems. These relationships are further explored in [1], and in a much deeper manner.

In the table of contents below, chapters and sections, marked by asterisks, can be skipped without loss of continuity.

## Reference

1. N. Merhav, Statistical physics and information theory. *Foundat. Trends Commun. Inf. Theor.* **6** (1–2), pp. 1–212, 2009.

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# Introduction

Statistical physics is a branch in physics which deals with systems with a huge number of particles (or any other elementary units). For example, *Avogadro's number*, which is about  $6 \times 10^{23}$ , is the number of molecules in 22.4 liters of ideal gas at standard temperature and pressure. Evidently, when it comes to systems with such an enormous number of particles, there is no hope to keep track of the physical state (e.g., position and momentum) of each and every individual particle by means of the classical methods in physics, that is, by solving a gigantic system of differential equations pertaining to Newton's laws for all particles. Moreover, even if those differential equations could have been solved somehow (at least approximately), the information that they would have given us would be virtually useless. What we normally really want to know about our physical system boils down to a fairly short list of *macroscopic* quantities, such as energy, heat, pressure, temperature, volume, magnetization, and the like. In other words, while we continue to use the well-known laws of physics, even the classical ones, we no longer use them in the ordinary manner that we have known from elementary physics courses. Instead, we think of the state of the system, at any given moment, as a realization of a certain *probabilistic ensemble*. This is to say that we approach the problem from a probabilistic (or a statistical) point of view. The beauty of statistical physics is that it derives the *macroscopic* theory of thermodynamics (i.e., the relationships between thermodynamic potentials, temperature, pressure, etc.) as *ensemble averages* that stem from this probabilistic *microscopic* theory, in the limit of an infinite number of particles, that is, the *thermodynamic limit*.

The purpose of this book is to teach statistical mechanics and thermodynamics, with some degree of orientation toward students in electrical engineering. The main body of the lectures is devoted to statistical mechanics, whereas much less emphasis is given to the thermodynamics part. In particular, the idea is to let the laws of thermodynamics to be obtained as conclusions from the derivations in statistical mechanics.

Beyond the variety of central topics in statistical physics that are important to the general scientific education of the electrical engineering student, special emphasis is

devoted to subjects that are vital to the engineering education concretely. These include, first of all, quantum statistics, like the Fermi–Dirac distribution, as well as diffusion processes, which are both fundamental for understanding semiconductor devices. Another important issue for the electrical engineering student is to understand mechanisms of noise generation and stochastic dynamics in physical systems, most notably, in electric circuitry. Accordingly, the fluctuation–dissipation theorem of statistical mechanics, which is the theoretical basis for understanding thermal noise processes and physical systems, is presented from the standpoint of a system with an input and output, in a way that would be understandable and useful for an engineer, and well related to other courses in the undergraduate curriculum of electrical engineering, like courses on random processes. This engineering perspective is not available in standard physics textbooks. The quantum regime, in this context, is important and hence provided as well. Finally, we touch upon some relationships between statistical mechanics and information theory, and demonstrate how the statistical–mechanical approach can be useful for the study of information theoretic problems. These relationships are further explored, and in a much deeper manner, in [1].

Most of the topics in this book are covered on the basis of several other well-known books on statistical mechanics. However, several perspectives and mathematical derivations are original and new (to the best of the author’s knowledge). The book includes fairly many examples, exercises, and figures, which will hopefully help the student to grasp the material better.

It is assumed that the reader has prior background in the following subjects: (i) elementary calculus and linear algebra, (ii) basics of quantum mechanics, and (iii) fundamentals of probability theory. Chapter 7 assumes also basic background in signals-and-systems theory, as well as the theory of random processes, including the response of linear systems to random input signals.

## Reference

1. N. Merhav, Statistical physics and information theory. *Foundat. Trends Commun. Inf. Theor.* **6** (1–2), pp. 1–212, 2009.