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Graduate Texts in Physics

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Nhan Phan-Thien · Nam Mai-Duy

Understanding Viscoelasticity

An Introduction to Rheology

Third Edition

 Springer

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Preface to the Third Edition

In this third edition, we have updated on the Dissipative Particle Dynamics method with recent results and provided some MATLAB programs for students to try out in fluid/flow modelling. In addition, a solution manual is provided, to guide students in their attempts at the questions in the book. The book itself remains compact, but sufficient in content for a first-year graduate module.

We sincerely thank our wives, Kim-Thoa (N. Phan-Thien) and Oanh (N. Mai-Duy) for their encouragement and their unfailing support—this is their achievement as well as ours!

Singapore
Toowoomba, Australia

Nhan Phan-Thien
Nam Mai-Duy

Preface to the Second Edition

In this second edition, typographical errors brought about by the conversion process to L^AT_EX were corrected; my gratitude went to Brittany Bannish (University of Utah) for painstakingly going through the first edition. My main aim in revising this is to produce a still compact book, sufficient at the level of first-year graduate course for those who wish to understand viscoelasticity, and to embark in modelling viscoelastic multiphase fluids. To this end, I have decided to introduce a new chapter on Dissipative Particle Dynamics (DPD), which I believe is relevant in modelling complex-structured fluids. All the basic ideas in DPD are reviewed, with some sample problems to illustrate the methodology. My gratitude goes to A*STAR, the Agency for Science, Technology and Research, for funding Multiphase Modelling Projects, and Prof. Khoo Boo Cheong, a colleague and above all a friend, for his support, which made the writing of Chapter 9 possible. I wish to acknowledge Prof. Mai-Duy Nam and Dr. Pan Dingyi, for their contributions to the DPD research, and Prof. Yu Shaozheng, for his comments on the revised book. Lastly, my humble thanks to the continuing support and constant encouragement of my wife, Kim-Thoa—without her capable hands, normal daily tasks would be impossible, let alone revising this book! It has been good for me to go through this revision, and I sincerely hope that the readers find the book useful in their research works.

Singapore
June 2012

Nhan Phan-Thien

Preface to the First Edition

This book presents an introduction to viscoelasticity, in particular, to the theories of dilute polymer solutions and dilute suspensions of rigid particles in viscous and incompressible fluids. These theories are important, not just because they apply to practical problems of industrial interest, but because they form a solid theoretical base upon which mathematical techniques can be built, from which more complex theories can be constructed, to better mimic material behaviour. The emphasis is not on the voluminous current topical research, but on the necessary tools to understand viscoelasticity at a first-year graduate level.

Viscoelasticity, or Continuum Mechanics, or Rheology¹ (certainly not to be confused with Theology) is *the science of deformation and flow*. This definition was due to Bingham, who, together with Scott-Blair² and Reiner,³ helped form The Society of Rheology in 1929. Rheology has a distinguished history involving high-profile scientists. The idea that everything has a timescale and that if we are prepared to wait long enough, then everything will flow was known to the Greek philosopher Heraclitus, and prior to him, to the Prophetess Deborah—*The Mountains Flowed Before The Lord*.⁴ Not surprisingly, the motto of the Society of Rheology is *παντα ρει* (everything flows), a saying attributed to Heraclitus.

From the rheological viewpoint, there is no clear distinction between solid and liquid; it is a matter between the relative timescale T of the experiment to the timescale τ of the material concerned. The timescale ratio, $De = \tau/T$ is called the *Deborah number*. If this ratio is negligibly small, then one has a viscous fluid (more precise definition later), and if it is large, a solid, and in-between, a viscoelastic liquid. The timescale of the fluid varies considerably, from 10^{-13} s for water, to a

¹This word was coined by E.C. Bingham (1878–1946), Professor of Chemistry at Lafayette College, Pennsylvania. The Bingham fluid is named after him.

²G.W. Scott Blair (1902–1987), Professor of Chemistry at the University of Reading. His main contributions were in biorheology.

³M. Reiner (1886–1976), Professor of Mathematics at the Technion University of Haifa, Israel. He is remembered for contributing to the Reiner–Rivlin fluid.

⁴The Book of Judges.



The logo of the Society of Rheology

few milliseconds for automotive oils, to minutes for polymer solutions and to hours for melts and soft solids.

Graduate students of Rheology naturally have the unenviable task of walking the bridge between solid mechanics and fluid mechanics, and at the same time trying to grasp the more significant and relevant concepts. They often find it hard (at least for me, during my graduate days) to piece together useful information from several comprehensive monographs and published articles on this subject. This set of lectures is an attempt to address this problem—it contains the necessary tools to understand viscoelasticity but does not insist on giving the latest piece of information on the topic.

The book starts with an introduction to the basic tools from tensor and dyadic analysis. Some authors prefer Cartesian tensor notation, others, dyadic notation. We use both notations, and they will be summarised here. Chapter 2 is a review of non-Newtonian behaviour in flows; here, the elasticity of the liquid and its ability to support large tension in stretching can be responsible for variety of phenomena, sometimes counter-intuitive. Kinematics and the equations of balance are discussed in detail in Chapter 3, including the finite strain and Rivlin–Ericksen tensors. In Chapter 4, some classical constitutive equations are reviewed, and the general principles governing the constitutive modelling are outlined. In this Chapter, the order fluid models are also discussed, leading to the well-known result that the Newtonian velocity field is admissible to a second-order fluid in plane flow. Chapter 5 describes some of the popular engineering inelastic and the linear elastic models. The inelastic models are very useful in shear-like flows where viscosity/shear rate relation plays a dominant role. The linear viscoelastic model is a limit of the simple fluid at small strain—any model must reduce to this limit when the strain amplitude is small enough. In Chapter 6, we discuss a special class of flows known as viscometric flows in which both the kinematics and the stress are fully determined by the flow, irrespective of the constitutive equations. This class of flows is equivalent to the simple shearing flow. Modelling techniques for polymer solutions are discussed next in Chapter 7. Here one has a set of stochastic differential equations for the motion of the particles; the random excitations come from a white noise model of the collision between the solvent molecules and the particles. It is our belief that a relevant model should come from the microstructure; however, when the microstructure is so complex that a detailed model is not tractable, elements of continuum model should be brought in. Finally, an introduction to suspension mechanics is given in Chapter 8. I have deliberately left out a number of topics: instability, processing flows, electro-rheological fluids, magnetised fluids and

viscoelastic computational mechanics. It is hoped that the book forms a good foundation for those who wish to embark on the Rheology path.

This has been tested out in a one-semester course in Viscoelasticity at the National University of Singapore. It is entirely continuous-assessment based, with the assignments graded at different difficulty levels to be attempted—solving problems is an indispensable part of the education process. A good knowledge of fluid mechanics is helpful, but it is more important to have a solid foundation in Mathematics and Physics (Calculus, Linear Algebra, Partial Differential Equations), of a standard that every one gets in the first two years in an undergraduate Engineering curriculum.

I have greatly benefitted from numerous correspondence with my academic brother, Prof. Raj Huilgol, and my mentor, Prof. Roger Tanner. Prof. Jeff Giacomin read the first draft of this; his help is gratefully acknowledged.

Singapore
February 2002

Nhan Phan-Thien

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