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To the memory of my mother, Marjorie Rose (1914–2012); and to the future of Shirley and Anna.

Preface

This book represents an account of some parts of operator theory, developed mainly since the 1980s whose problems have their roots in quantum theory. The research presented is in non-commutative operator approximation theory or, to use Halmos' terminology, in operator approximants. The crucial concept of approximant is explicated in Chap. 1 ("What this book is about") where the range of problems is outlined. The setting is mainly, but not exclusively, the Von Neumann-Schatten classes $\mathcal{C}_p$.

Thus, quantum chemistry—approximating a Hamiltonian and the Lowdin orthogonalization—precipitates Chap. 3 ("Self-adjoint and positive approximants") and Chap. 6 ("Unitary, isometric and partially isometric approximants") respectively. The commutation relation of quantum mechanics precipitates Chap. 4 ("Commutator approximants").

I have tried—by some necessary simplification—to present the quantum theory background as self-contained. The book therefore assumes no scientific knowledge on the part of the reader. In any case, if the reader is interested in the mathematics alone then she or he could skip the quantum theory motivational sections; but that would be to miss some of the interest.

Obviously, this book presents the necessary mathematical machinery to tackle the various approximant problems. Specifically, Chap. 2 states the Aiken, Erdos and Goldstein result in differentiating the map $\mathcal{C}_p \to \mathbb{R}^+$ given by $X \mapsto \|X\|_p^p$, crucial for most of this work; and Chap. 5 develops the material on spectral approximants required in Chap. 6.

Chapter 2 onward come equipped with a set of exercises whose purpose is to extend, in various directions, the material presented in the body of the chapter. I strongly advise the reader to tackle these exercises since they will, as it were, enable the reader to actively participate in the content of this work. Solutions of many of the exercises can be found in the various papers discussed in the "Notes" with which each chapter concludes.

The reader of this book is expected to have a background in Hilbert space operator theory approaching that of Halmos' marvellous "A Hilbert Space Problem

Book" [23] (to which reference is frequently made). For such a reader this book is suitable for study at postgraduate level.

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Some of this work originated, long ago, in my Ph.D thesis. I thank Dr. John Erdos for the help he so freely and generously gave me as my Ph.D supervisor.

London, UK Philip J. Maher
March 2016

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