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James Lottes

Towards Robust Algebraic Multigrid Methods for Nonsymmetric Problems

Doctoral Thesis accepted by
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Supervisor's Foreword

Multigrid methods are a central methodology in Scientific Computing. As solvers (and preconditioners) for self-adjoint and positive definite elliptic partial differential equation problems they provide linear equation solution algorithms of optimal computational complexity. For other symmetric matrix systems they are an important computational technology, with both the classical geometric multilevel methods and the more recent algebraic multigrid algorithms finding widespread application in many areas of science and engineering. For non-self-adjoint/non-symmetric problems, however, the design of effective multigrid methods and their understanding has lagged far behind.

This thesis lays the theoretical foundation—through the introduction of the new concept of the 'form absolute value' of a matrix—and derives practical non-symmetric algebraic multigrid approaches with guaranteed convergence behaviour. The approach here is completely new; it generalises in a natural way the usual development for self-adjoint problems to which it reduces for symmetric problems.

This thesis represents a significant breakthough in terms of the new concepts and understanding it introduces. It opens the door for further development of highly effective and efficient algebraic multigrid algorithms for non-symmetric problems.

Oxford, UK
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Prof. Andrew Wathen

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