

# Use R!

## Series Editors

Robert Gentleman   Kurt Hornik   Giovanni Parmigiani

More information about this series at <http://www.springer.com/series/6991>

# Use R!

---

Recently Published in Use R!

*Boehmke*: Data Wrangling with R

*Wickham*: ggplot2

*Moore*: Applied Survival Analysis Using R

*Luke*: A User's Guide to Network Analysis in R

*Monogan*: Political Analysis Using R

*Cano/M. Moguerza/Prieto Corcoba*: Quality Control with R

*Schwarzer/Carpenter/Rücker*: Meta-Analysis with R

*Gondro*: Primer to Analysis of Genomic Data Using R

*Chapman/Feit*: R for Marketing Research and Analytics

*Willekens*: Multistate Analysis of Life Histories with R

*Cortez*: Modern Optimization with R

*Kolaczyk/Csárdi*: Statistical Analysis of Network Data with R

*Swenson/Nathan*: Functional and Phylogenetic Ecology in R

*Nolan/Temple Lang*: XML and Web Technologies for Data Sciences with R

*Nagarajan/Scutari/Lèbre*: Bayesian Networks in R

*van den Boogaart/Tolosana-Delgado*: Analyzing Compositional Data with R

*Bivand/Pebesma/Gómez-Rubio*: Applied Spatial Data Analysis with R

(2nd ed. 2013)

*Eddelbuettel*: Seamless R and C++ Integration with Rcpp

*Knoblauch/Maloney*: Modeling Psychophysical Data in R

Stefano M. Iacus · Nakahiro Yoshida

# Simulation and Inference for Stochastic Processes with YUIMA

A Comprehensive R Framework for SDEs  
and Other Stochastic Processes

Stefano M. Iacus  
Department of Economics, Management  
and Quantitative Methods  
University of Milan  
Milan  
Italy

Nakahiro Yoshida  
Graduate School of Mathematical  
Sciences  
University of Tokyo  
Tokyo  
Japan

ISSN 2197-5736

ISSN 2197-5744 (electronic)

Use R!

ISBN 978-3-319-55567-6

ISBN 978-3-319-55569-0 (eBook)

<https://doi.org/10.1007/978-3-319-55569-0>

Library of Congress Control Number: 2017934627

© Springer International Publishing AG, part of Springer Nature 2018

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by the registered company Springer International Publishing AG part of Springer Nature

The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*To Maite, Lucy and Ludo,  
to whom I wish to find what gives  
their life true meaning and purpose  
and to Tizy,  
whose heart I met late in my life,  
but, thanks God, not too late!*

Stefano M. Iacus

# Preface

Statistics for stochastic processes is rapidly developing. It forms a branch of mathematical sciences, spreading over theoretical statistics, probability theory, software development and real data analysis. Since a general theoretical framework of statistical inference for stochastic processes was recently established, statistical inference has been applicable to various stochastic systems and its scope is expanding more and more from ergodic to nonergodic processes, from low-frequency regular to high-frequency irregular sampling schemes, from linear to nonlinear models, and so on.

The formulas provided by the theory are often fairly complicated, and it makes it difficult for nonexperts to use them in their own fields. For example, an asymptotic expansion formula derived by the Malliavin calculus involves hundreds of terms, the Bayesian estimator theoretically validated recently needs modern MCMC methods for computation in practice, and some random number generators for simulation of Lévy-driven stochastic differential equations use quite sophisticated algorithms. Software implementation is an issue in such circumstances.

YUIMA is a computational framework for statistical analysis and simulation for stochastic processes, especially objects described in terms of the stochastic analysis. YUIMA is designed to realize a circle of data analysis, modelling, fitting, simulation, and prediction. Through YUIMA, the user enjoys easily, without depending on his/her expertise, the latest developments in theoretical statistics for stochastic processes.

The YUIMA Project was launched by Stefano Maria Iacus and Nakahiro Yoshida, respectively, an R guru and a dreamer, after a three-person discussion around 2005 set by Masayuki Uchida. Supported by Japan Science and Technology Agency PRESTO (2007–2011), the project implemented a basic structure on R and extended by inviting Hideitsu Hino, Hiroki Masuda, Yasutaka Shimuzu, Kengo Kamatani, Alexandre Brouste, Masaaki Fukasawa and Teppei Ogihara. Hino with the Waseda University team was quite active in programming many YUIMA functions. Collaboration with Kenji Kashiwakura and Kentaro Hoshi with their NS Solutions team in 2012–2016 is acknowledged. The YUIMA Project got new members Yuta Koike, Ryosuke Nomura, Lorenzo Mercuri, Yusuke Shimizu,

Shoichi Eguchi, Yuma Uehara, Yuto Yoshida, Emanuele Guidotti and many other young people. Most of them are mathematical statisticians, and this is the reason why the functions of YUIMA are structurally designed with rigorous mathematical backing. Presently, the YUIMA Project (YUIMA III) is supported by Japan Science and Technology Agency CREST led by Prof. Takashi Tsuboi. Special thanks go to Prof. Shigeo Kusuoka for his great support for statistics in mathematics. The authors also thank Mrs. Sayako Takehara and Miss Homare Yoshihira for their help as the secretaries of the laboratory.

We also need to thank MIUR—Ministero dell’Istruzione, dell’Università e della Ricerca, Grant: PRIN 2009JW2STY, for supporting the early work of Lorenzo Mercuri, Emanuele Guidotti and the first author on this project.

Milan, Italy  
Tokyo, Japan  
February 2018

Stefano M. Iacus  
Nakahiro Yoshida

# Contents

## Part I The YUIMA Framework

<b>1</b>	<b>The YUIMA Package</b>	3
1.1	Overview of the Project	3
1.2	Who Should Read This Book?	4
1.3	Structure of the Book	4
1.4	How to Get the R Code for This Book	5
1.5	Main Contribution to the Yuima Package	5
1.6	Further Developments of Yuima Package	6
1.7	Things to Know About R	7
1.7.1	How to Get R	7
1.7.2	R and S4 Objects	7
1.8	The Yuima Package	11
1.8.1	How to Obtain the Package	11
1.8.2	The Main Object and Classes	12
1.8.3	The <code>yuima.model</code> Class	15
1.9	On Model Specification	16
1.9.1	Basic Model Specification	17
1.9.2	User-Specified State and Time Variables	18
1.9.3	Specification of Parametric Models	19
1.10	Basic Facts on Simulation	20
1.10.1	Customization of Simulation Arguments	20
1.10.2	Simulation of Models with User-Specified Notation	23
1.10.3	Simulation of Parametric Models	24
1.11	Sampling and Simulate	25
1.11.1	Sampling and Subsampling	27
1.12	How to Make Data Available into a <code>yuima</code> Object	33
1.12.1	Getting Data from Data Providers	37
1.13	How to Extract Data from a <code>yuima</code> Object	39



1.14	Time Series Classes, Time Data and Time Stamps . . . . .	40
1.14.1	Review of Some Time Series Objects in R . . . . .	40
1.14.2	How to Handle Real Time Stamps . . . . .	47
1.14.3	Dates Manipulation . . . . .	50
1.14.4	Using Dates to Index Time Series . . . . .	52
1.14.5	Joining Two or More Time Series . . . . .	54
1.14.6	Subsetting a Time Series . . . . .	59
1.15	Miscellanea . . . . .	63
1.15.1	From Yuima to L <sup>A</sup> T <sub>E</sub> X . . . . .	63
1.15.2	The Yuima GUI . . . . .	65

## Part II Models and Inference

<b>2</b>	<b>Diffusion Processes . . . . .</b>	<b>69</b>
2.1	Model Specification . . . . .	69
2.1.1	Ornstein–Uhlenbeck (OU) . . . . .	72
2.1.2	Geometric Brownian Motion (gBm) . . . . .	72
2.1.3	Vasicek Model (VAS) . . . . .	73
2.1.4	Constant Elasticity of Variance (CEV) . . . . .	73
2.1.5	Cox–Ingersoll–Ross Process (CIR) . . . . .	73
2.1.6	Chan–Karolyi–Longstaff–Sanders Process (CKLS) . . . . .	74
2.1.7	Hyperbolic Diffusion Processes . . . . .	74
2.2	More About Simulation . . . . .	78
2.3	Multidimensional Processes . . . . .	80
2.3.1	The Heston Model . . . . .	82
2.4	Parametric Inference . . . . .	84
2.4.1	Quasi-maximum Likelihood Estimation . . . . .	85
2.4.2	Adaptive Bayes Estimation . . . . .	87
2.5	Example of Real Data Estimation for gBm . . . . .	91
2.6	Example of Real Data Estimation for CIR . . . . .	93
2.7	Hypotheses Testing . . . . .	96
2.8	AIC Model Selection . . . . .	100
2.8.1	An Example of AIC Model Selection for Exchange Rates Data . . . . .	101
2.9	LASSO Model Selection . . . . .	103
2.9.1	An Example of Lasso Model Selection for Interest Rates Data . . . . .	106
2.10	Change Point Estimation . . . . .	108
2.10.1	Example of Volatility Change Point Estimation for Two-Dimensional SDEs . . . . .	109
2.10.2	An Example of Two-Stage Estimation . . . . .	112
2.10.3	Example of Volatility Change Point Estimation in Real Data . . . . .	114

- 2.11 Asynchronous Covariance Estimation . . . . . 115
  - 2.11.1 Example: Data Generation and Estimation by yuima Package . . . . . 117
  - 2.11.2 Asynchronous Estimation for Nonlinear Systems . . . . . 121
  - 2.11.3 Other Covariance Estimators . . . . . 122
- 2.12 Lead–Lag Estimation . . . . . 123
  - 2.12.1 Application of the Lead–Lag Estimator to Real Data . . . 128
- 2.13 Asymptotic Expansion . . . . . 130
  - 2.13.1 Asymptotic Expansion for General Stochastic Processes . . . . . 134
- 3 Compound Poisson Processes . . . . . 137**
  - 3.1 Inhomogeneous Compound Poisson Process . . . . . 140
    - 3.1.1 Linear Intensity Function . . . . . 141
    - 3.1.2 The Weibull Model . . . . . 141
    - 3.1.3 The Exponentially Decaying Intensity Model . . . . . 142
    - 3.1.4 Modulated and Periodical Intensity Model . . . . . 142
    - 3.1.5 Frequency Modulation Model . . . . . 144
  - 3.2 Multidimensional Compound Poisson Processes . . . . . 144
    - 3.2.1 Multivariate Gaussian Jumps . . . . . 145
    - 3.2.2 User-Specified Jump Distribution . . . . . 146
  - 3.3 Estimation . . . . . 148
    - 3.3.1 Compound Poisson Process with Gaussian Jumps . . . . . 148
    - 3.3.2 NIG Compound Poisson Process . . . . . 150
    - 3.3.3 Exponential Jump Compound Poisson Process . . . . . 151
    - 3.3.4 The Weibull Compound Poisson Process . . . . . 152
    - 3.3.5 Modulated and Periodical Intensity Model . . . . . 153
- 4 Stochastic Differential Equations Driven by Lévy Processes . . . . . 155**
  - 4.1 Lévy Processes . . . . . 155
    - 4.1.1 Infinitely Divisible Distributions . . . . . 156
    - 4.1.2 Infinite Divisible Distributions, Lévy Processes, Lévy-Itô Decomposition . . . . . 157
  - 4.2 Wiener Process . . . . . 158
  - 4.3 Compound Poisson Process . . . . . 158
  - 4.4 Gamma Process and Its Variants . . . . . 159
    - 4.4.1 Gamma Process . . . . . 159
    - 4.4.2 Variance Gamma Process . . . . . 160
    - 4.4.3 Bilateral Gamma Process . . . . . 161
    - 4.4.4 Simulation of Gamma Processes . . . . . 162
  - 4.5 Generalized Tempered Stable Process, Tempered  $\alpha$  Stable Process, CGMY Process, Positive Tempered Stable Process . . . . . 165
  - 4.6 Inverse Gaussian Process . . . . . 165
  - 4.7 Increasing Stable Process . . . . . 169

- 4.8 Subordination . . . . . 171
  - 4.8.1 Definition . . . . . 171
  - 4.8.2 Compound Poisson Process by Subordination . . . . . 171
  - 4.8.3 Subordination of a Wiener Process with Drift . . . . . 172
  - 4.8.4 Variance Gamma Process with Drift . . . . . 173
  - 4.8.5 Normal Inverse Gaussian Process . . . . . 175
  - 4.8.6 Normal Tempered Stable Process . . . . . 179
- 4.9 Stable Process . . . . . 181
- 4.10 Generalized Hyperbolic Processes . . . . . 184
  - 4.10.1 Generalized Inverse Gaussian Distribution . . . . . 184
  - 4.10.2 Generalized Inverse Gaussian Process  
and Generalized Hyperbolic Process . . . . . 185
  - 4.10.3 GH Distributions . . . . . 186
  - 4.10.4 Subclasses of the GH Distributions . . . . . 187
- 4.11 Stochastic Differential Equation Driven by Lévy Processes  
and Their Simulation . . . . . 188
  - 4.11.1 Semimartingale . . . . . 188
  - 4.11.2 Stochastic Differential Equations . . . . . 190
  - 4.11.3 Compound Poisson Driving Processes . . . . . 190
  - 4.11.4 Driving Processes of code Type . . . . . 192
- 4.12 Estimation . . . . . 196
  - 4.12.1 Estimation of Jump-Diffusion Processes . . . . . 196
  - 4.12.2 Estimation of Exponential Lévy Processes . . . . . 199
- 4.13 Bessel Function of the Third Kind . . . . . 201
- 5 Stochastic Differential Equations Driven by the Fractional  
Brownian Motion . . . . . 203**
  - 5.1 Model Specification . . . . . 203
  - 5.2 Simulation of the Fractional Gaussian Noise . . . . . 205
    - 5.2.1 Cholesky Method . . . . . 206
    - 5.2.2 Wood and Chan Method . . . . . 206
  - 5.3 Simulation of Fractional Stochastic Differential Equations . . . . . 207
  - 5.4 Parametric Inference for the fOU . . . . . 208
    - 5.4.1 Estimation of the Hurst Exponent and the Diffusion  
Coefficient via Quadratic Generalized Variations . . . . . 209
    - 5.4.2 Estimation of the Drift Parameter . . . . . 211
  - 5.5 An Example on Climate Change Data . . . . . 212
- 6 CARMA Models . . . . . 215**
  - 6.1 Lévy-Driven CARMA Models . . . . . 215
  - 6.2 CARMA Model Specification . . . . . 217
    - 6.2.1 The `yuima.carma-class` . . . . . 217
  - 6.3 CARMA( $p,q$ ) Model Estimation . . . . . 220

- 6.4 Examples of Lévy-driven CARMA( $p,q$ ) Models . . . . . 222
  - 6.4.1 Compound Poisson CARMA(2,1) Process . . . . . 222
  - 6.4.2 Variance Gamma CARMA(2,1) Process . . . . . 224
  - 6.4.3 Normal Inverse Gaussian CARMA(2,1) Process . . . . . 226
- 6.5 Application to the VIX Index . . . . . 229
- 7 COGARCH Models . . . . . 237**
  - 7.1 General Order ( $p,q$ ) Model . . . . . 239
    - 7.1.1 How to Input a COGARCH( $p,q$ ) Model in *yuima* . . . . . 240
    - 7.1.2 Stationarity Conditions . . . . . 241
  - 7.2 Simulation Schemes . . . . . 244
  - 7.3 Generalized Method of Moments Estimation . . . . . 248
    - 7.3.1 Moments Matching Step . . . . . 248
    - 7.3.2 Lévy Distribution Estimation . . . . . 250
  - 7.4 Quasi-maximum Likelihood Estimation . . . . . 251
  - 7.5 Relationship Between GARCH(1,1) and COGARCH(1,1) . . . . . 253
  - 7.6 Application to Real Data . . . . . 254
- References . . . . . 257**
- Index . . . . . 265**