

UNITEXT for Physics

Series editors

Michele Cini, Roma, Italy

Attilio Ferrari, Torino, Italy

Stefano Forte, Milano, Italy

Guido Montagna, Pavia, Italy

Oreste Nicrosini, Pavia, Italy

Luca Peliti, Napoli, Italy

Alberto Rotondi, Pavia, Italy

Paolo Biscari, Milano, Italy

Nicola Manini, Milano, Italy

Morten Hjorth-Jensen, Oslo, Norway

More information about this series at <http://www.springer.com/series/13351>

Emilio d'Emilio · Luigi E. Picasso

Problems in Quantum Mechanics

with Solutions

Second Edition

 Springer

Emilio d'Emilio
Dipartimento di Fisica
Università di Pisa
Pisa
Italy

Luigi E. Picasso
Dipartimento di Fisica
Università di Pisa
Pisa
Italy

ISSN 2198-7882

UNITEXT for Physics

ISBN 978-3-319-53266-0

DOI 10.1007/978-3-319-53267-7

ISSN 2198-7890 (electronic)

ISBN 978-3-319-53267-7 (eBook)

Library of Congress Control Number: 2017932006

1st edition: © Springer-Verlag Italia 2011

2nd edition: © Springer International Publishing AG 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature

The registered company is Springer International Publishing AG

The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface to the Second Edition

The present edition of the book follows the first one by about five years.

Apart from the corrections of a few minor misprints and the updating of the precise values of the fundamental constants (and related problems 2.2, 2.3), the main reason for presenting this second edition lies in the fact that in 2015 also the companion textbook “Lectures in Quantum Mechanics” by one of us (LEP) was published by Springer: such textbook contains an extra chapter, ‘Elementary Theory of Scattering’, a subject not dealt with by the first edition of “Problems”. As a consequence the authors felt the necessity to fill the gap.

Indeed the present edition has one more chapter, the final one, with 21 further problems. Some of them are simple. Others instead, much in the spirit of the rest of the book, offer – in pills – important subjects that often are just hinted at in textbooks. In the end the chapter proposes a number of concrete examples that provide a sort of guided tour through the main topics in potential scattering theory.

Concerning the last chapter, we find appropriate to repeat here the recommendation given in the preface to the first edition: the student should try all the problems hard and should not feel discomforted if he or she will have to resort to the solutions – he or she will, in any event, learn something more.

Pisa, December 2016

Emilio d’Emilio
Luigi E. Picasso

Preface to the First Edition

This book stems from the experience the authors acquired by teaching Quantum Mechanics over more than two decades.

The necessity of providing students with abundant and understandable didactic material – i.e. exercises and problems good for testing “in real time” and day by day their comprehension and mastery of the subject – confronted the authors with the necessity of adapting and reformulating the vast number of problems available from the final examinations given in previous years. Indeed those problems, precisely because they were formulated as final exam problems, were written in a language appropriate for the student who is already a good step ahead in his preparation, not for the student that, instead, is still in the “middle of the thing”.

Imagining that the above necessity might be common to colleagues from other Departments and prompted also by the definite shortage, in the literature, of books written with this intent, we initially selected and ordered the 242 problems presented here by sticking to the presentation of Quantum Mechanics given in the textbook “Lezioni di Meccanica Quantistica” (ETS, Pisa, 2000) by one of us (LEP).

Over time, however, our objective drifted to become making the present collection of problems more and more autonomous and independent of any textbook. It is for this reason that certain technical subjects – as e.g. the variational method, the virial theorem, selection rules etc. – are exposed in the form of problems and subsequently taken advantage of in more standard problems devoted to applications.

The present edition – the first in English – has the advantage over the Italian one [“Problemi di Meccanica Quantistica” (ETS, Pisa 2003, 2009)] that all the material has by now been exhaustively checked by many of our students, which has enabled us to improve the presentation in several aspects.

A comment about the number of proposed problems: it may seem huge to the average student: almost certainly not all of them are necessary to have a satisfactory insight into Quantum Mechanics. However it may happen –

particularly to the student who will take further steps towards becoming a professional physicist – that he or she will have to come back, look at, and even learn again certain things . . . well, we do not hide our intent: this book should not be just for passing exams but, possibly, for life.

Here are a few further comments addressed to students who decide to go through the book. Firstly, some of the problems (also according to our students) are easy, standard, and just recall basic notions learned during the lectures. Others are not so. Some of them are definitely difficult and complex, mainly for their conceptual structure. However, we had to put them there, because they usually face (and we hope clarify) questions that are either of outstanding importance or rarely treated in primers. The student should nonetheless try them using all his or her skill, and not feel frustrated if he or she cannot completely solve them. In the latter case the solution can be studied as a part of a textbook: the student will anyhow learn something new. Second, despite our effort, it may happen (seldom, we hope) that a symbol used in the text has not been defined in the immediately previous lines: it can be found in the Appendices. Our claim also is that all the problems can be solved by simple elementary algebra: the more complicated, analytic part of the calculation – when present – should take advantage of the proposed suggestions (e.g. any awkward, or even elementary, integral supposed to appear in the solution is given in the text) and should be performed in such a way as to reduce all the formulae to those given in the Appendices.

A last comment concerns the way numerical calculations are organized, particularly in the first chapters. We have written dimensionless numbers as the ratio of known quantities, e.g. two energies, two masses . . . (so that a better dimensional control of what is being written is possible at a glance and at any step of the calculation – a habit the student should try hard to develop) and we have used the numerical values of these known quantities given in Appendix A: this is quicker and safer than resorting to the values of the fundamental constants.

Among the many persons – students, colleagues, families – who helped us over years in this work, three played a distinguished role. We are thankful to Pietro Menotti, maybe the only one of our colleagues with a more long-lasting didactic experience of the subject, for the very many comments and suggestions and for having been for one of us (EdE) a solid reference point along the twenty years of our didactic collaboration. Stephen Huggett helped us with our poor English. Bartolome Alles Salom, in addition to having gone through the whole book with an admirable painstaking patience, has a major responsibility for the appearance of the present English edition, having driven and convinced us with his enthusiasm to undertake this job.

Of course all that could have (and has not yet) been improved is the authors' entire responsibility.

Pisa, May 2011

*Emilio d'Emilio
Luigi E. Picasso*

Contents

1 Classical Systems

Atomic models; radiation; Rutherford scattering; specific heats; normal modes of vibration.

Problems	1
Solutions	5

2 Old Quantum Theory

Spectroscopy and fundamental constants; Compton effect; Bohr–Sommerfeld quantization; specific heats; de Broglie waves.

Problems	13
Solutions	20

3 Waves and Corpuscles

Interference and diffraction with single particles; polarization of photons; Malus' law; uncertainty relations.

Problems	29
Solutions	37

4 States, Measurements and Probabilities

Superposition principle; observables; statistical mixtures; commutation relations.

Problems	47
Solutions	53

5 Representations

Representations; unitary transformations; von Neumann theorem; coherent states; Schrödinger and momentum representations; degeneracy theorem.

Problems	63
Solutions	74

6 One-Dimensional Systems

Nondegeneracy theorem; variational method; rectangular potentials; transfer matrix and S -matrix; delta potentials; superpotential; completeness.

Problems	93
Solutions	107

7 Time Evolution

Time evolution in the Schrödinger and Heisenberg pictures; classical limit; time reversal; interaction picture; sudden and adiabatic approximations.

Problems	139
Solutions	149

8 Angular Momentum

Orbital angular momentum: states with $l = 1$ and representations; rotation operators; spherical harmonics; tensors and states with definite angular momentum ($l = 1, l = 2$).

Problems	167
Solutions	173

9 Changes of Frame

Wigner's theorem; active and passive point of view; reference frame: translated, rotated; in uniform motion; in free fall, rotating.

Problems	185
Solutions	190

10 Two and Three-Dimensional Systems

Separation of variables; degeneracy theorem; group of invariance of the two-dimensional isotropic oscillator.

Problems	199
Solutions	204

11 Particle in Central Field

Schrödinger equation with radial potentials in two and three dimensions; vibrational and rotational energy levels of diatomic molecules.

Problems	213
Solutions	219

12 Perturbations to Energy Levels

Perturbations in one-dimensional systems; Bender–Wu method for the anharmonic oscillator; Feynman–Hellmann and virial theorems; “no-crossing theorem”; external and internal perturbations in hydrogen-like ions.

Problems	231
Solutions	243

13 Spin and Magnetic Field

Spin $\frac{1}{2}$; Stern and Gerlach apparatus; spin rotations; minimal interaction; Landau levels; Aharonov–Bohm effect.

Problems	263
Solutions	272

14 Electromagnetic Transitions

Coherent and incoherent radiation; photoelectric effect; transitions in dipole approximation; angular distribution and polarization of the emitted radiation; life times.

Problems	283
Solutions	291

15 Composite Systems and Identical Particles

Rotational energy levels of polyatomic molecules; entangled states and density matrices; singlet and triplet states; composition of angular momenta; quantum fluctuations; EPR paradox; quantum teleportation.

Problems 301
 Solutions 309

16 Applications to Atomic Physics

Perturbations on the fine structure energy levels of the hydrogen atom; electronic configurations and spectral terms; fine structure; Stark and Zeeman effects; intercombination lines.

Problems 323
 Solutions 333

17 Elementary Potential Scattering

One-dimensional systems; time-delay; optical theorem; hard sphere; spherical barrier; spherical potential well; spherical Dirac delta-shell; resonances at low energies; bound states and virtual levels as poles of the *S*-matrix; Breit-Wigner formula; Jost functions; Levinson theorem; Ramsauer-Townsend effect; Yukawa potential; Bragg reflection; identical particles.

Problems 347
 Solutions 358

Appendix A Physical Constants 375
Appendix B Useful Formulae 377
Index 379