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Jian-Jun Xu

Interfacial Wave Theory of Pattern Formation in Solidification

Dendrites, Fingers, Cells and Free Boundaries

Second Edition

Springer

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I dedicate this book to my family, my wife Xu Chen, my three sons: Wen-Bo, Wen-Tao, Wen-Lei, as well as to all persons who have helped me in my life

Preface of the Second Edition

This is a new extended version of the book that follows after its first edition published in 1998. After then, my studies on interfacial pattern formation in solidification have been extended into much broader areas in material science. Most results of these studies were scattered, published in some popular journals in physics, material science and applied mathematics. It seems that now is a proper time to present a new, extended version of book in the framework of synergetic theory, that summarizes all these advances in terms of a unified analytical approach. The present book covers three types of prominent pattern formation phenomena that are commonly observed in various systems of growth and solidification in material science: the single free dendritic growth from undercooled melt or binary mixture; the spatially-periodic cellular growth and the spatially-periodic eutectic growth from binary mixtures in directional solidification. It is found that the interfacial wave (IFW) theory that was previously established for mainly describing free dendritic growth is applicable for describing other two types of pattern formation phenomena as well.

The important characteristics of this book in comparison with other books on similar subjects available in the literature are the followings. First of all the book aims at the outstanding real-world problem, rather than some artificial mathematical model problem. It starts from the description of the phenomenon observed in experiments, then by making all possible simplifications, formulates the justifiable mathematical model for the phenomenon based on the first physical principles without artificial hypotheses. This process leads to the mathematical problems under studies.

Secondly the book uses the unified asymptotic approaches to explore all these pattern formation systems, rather then using numerical simulations. The analytical formulas obtained can be used for better and deeper elucidating the essence of the phenomena, as well as the mathematical similarities between various systems of growth and solidification with interfacial energy.

Thirdly, the book emphasizes the profound understanding of underlying physical mechanisms of the phenomena and the roles played by each of physical parameters in these mechanisms. The book reveals the intrinsic similarity in the essence of the dynamic phenomena observed in a variety of growth systems. In this way, it exhibits the unification and harmonics of the physical world, as well as the power and beauty of the mathematics.

Finally, this book emphasizes the idea that the reliable theoretical results and conclusions must undergo rigour verifications of the experimental data and observations. Hence, we made every effort to conduct the quantitative comparisons between the mathematical results and available experimental data without involving free parameter, and examine the theoretical predictions via experimental evidence. In fact, we attempt to establish a predictive mathematical theory for the physical phenomena under studies.

In the book, the readers will be guided to deal with a complicated dynamic system by the following procedure: determining the basic steady states for the systems; performing the linear stability analysis; deriving the local dispersion relationship for local normal modes in the outer region; finding out the singularity of the normal mode solutions; deriving the inner solutions near the singular point; matching the inner solution with the outer solution and, eventually, solving the linear eigenvalue problem (EVP). This thinking path runs through the whole book.

This procedure leads to the analytical forms of the global modes and the corresponding quantization conditions for the spectra of eigenvalues. It is shown that the interfacial perturbed states in all the systems under the study can be considered as the same type of interfacial waves subjecting a cubic complex dispersion relationship. The linear perturbed systems generally allow the real eigenvalue spectrum and the complex eigenvalue spectrum. It is therefore derived that the limiting state of these systems is either steady or oscillating; all oscillatory patterns observed are a wave phenomenon controlled by essentially the same mechanism—the so-called global trapped wave mechanism; the pattern selected at the later stage of experiments under steady growth conditions is eventually either described by the most dangerous global neutral mode of the system, or described by the steady state in the global stable region bounded by the most dangerous global neutral curve in the state space.

By the chance of the present edition, I refined the mathematical derivations performed in the first edition and included more details of derivations. Some algebraic errors in the derivations found in in previous edition are corrected and related numerical results are re-calculated. The changes and corrections do not alter the qualitative behaviours of the solutions and conclusions, but induce some quantitative, numerical changes.

The book presents the works that lie on the bounder between mathematics, physics and material science. The interfacial pattern formation itself has been an important subject in the field of interdisciplinary non-linear science. The book describes how the applied mathematics is used to give insight into the problems of growth and material processing, and how the physical problems can lead to new mathematical problems. As a consequence, for the readers from the area of material science and engineering, a complete understanding of all of the mathematics in the book will require at least a solid undergraduate training in mathematics, whereas for the readers from area of mathematics, a solid background in general physics, thermodynamics and statistical physics is necessary. Moreover, to follow the book, the readers will be also required the basic understanding of ordinary and partial differential equations, stability, bifurcation, and basic dynamic system theory. In particular, the readers are required to be familiar with the asymptotic analysis, singular perturbation theory, multiple variable expansion methods, since the book makes extensively use of these methods. For the readers whose mathematical background is lack of these topics, we recommend them to consult the excellent classical books by Kevorkian and Cole (1996) and Holmes (2013).

The researchers, post-doctoral and graduate students in the fields of condensed matter physics, material science, applied mathematics, mechanical engineering, and chemical engineering will find this book useful and beneficial. The book can be also used as a reference book for teaching a graduate course of mathematical theory of crystal growth, solidification and material processing, or a graduate course of advanced topics in material science or in applied mathematics.

Finally, for the completion of this project, credit should be given to a number of people, such as Prof. Martin Glicksman, Prof. Yoshinori Furukawa, Prof. Alain Pocheau and their co-workers, from whom I received and used their significant experimental results and photos. I thank many colleagues and friends for their invaluable advice, encouragement and discussions during the years of carrying on this project. On this regard, Prof. Stephen H. Davis and Robert E. O'Malley should be especially mentioned. I very much thank my colleague, former graduate student, Prof. Yong-Qiang Chen for his valuable contributions on this project and his extensive numerical computation work in preparing this edition of the book. I am grateful to all members of my family for their long term of tolerance, understanding and support; the book can never be completed without their help.

Montreal, QC, Canada Jian-Jun Xu
November 2016

Preface of the First Edition

For the last several years, the study of interfacial instability and pattern formation phenomena has preoccupied many researchers in the broad area of nonlinear science. These phenomena occur in a variety of dynamical systems far from equilibrium. In many practically very important physical systems some fascinating patterns are always displayed at the interface between solid and liquid or between two liquids. Two prototypes of these phenomena are dendritic growth in solidification and viscous fingering in a Hele–Shaw cell. These two phenomena occur in completely different scientific fields, but both are described by similar nonlinear free boundary problems of partial-differential-equation systems; the boundary conditions on the interface for both cases contain a curvature operator involving the surface tension, which is nonlinear. Moreover, both cases raise the same challenging theoretical issues, interfacial instability mechanisms and pattern selection, and it is now found that these issues can be solved by the same analytical approach. Thus, these two phenomena are regarded as special examples of a class of nonlinear pattern formation phenomena in nature, and they are the prominent topics of the new interdisciplinary field of nonlinear science.

This research monograph is based on a series of lectures I have given at McGill University, Canada (1993–1994), Northwestern Polytechnical Institute, China (1994), Aachen University, Germany (1994), and the CRM summer school at Banff, Alberta, Canada (1995). I shall illustrate these phenomena, present the fundamental issues involved, and describe the recent theoretical developments. In particular, I shall discuss some long-standing problems in this field, such as the pattern selection principle and the interfacial instability mechanism, and systematically present a newly established, coherent, predictive theory of this subject: the interfacial wave theory.

This book will be useful for researchers, post-doctoral and graduate students in the fields of condensed matter physics, materials science, applied mathematics, mechanical engineering, and chemical engineering.

I am particularly indebted to Prof. J.D. Cole for teaching me the asymptotic analysis and various perturbation methods. I also thank him and Prof. M.E. Glicksman for introducing me to this challenging and exciting field and for their constant discussions and advice. During the past decade, I have also received invaluable help and advice from, and had discussions with, many other people in my investigation of this subject, as well as in the preparation of this book. This book could not have been written without their help; among them, Prof. C.C. Lin at MIT, Prof. S.H. Davis at Northwestern University, Prof. R.E. O'Malley at the University of Washington, and Dr. John R. Ockendon at Oxford University should be especially mentioned.

I thank my graduate student, Mr. Dong-Sheng Yu very much for carrying out the large amount of numerical computations and for preparing the figures. I also thank him and my graduate student, Mr. Mikhail Kharenko, for the careful verification of all the formulae. I also thank Miss Katina Michelis, a graduate student in my course, 'Topics in Applied Mathematics', for carefully proof-reading and checking the formulae in the manuscript. I would like to thank the production team at Springer-Verlag for their excellent editing work.

Montreal, QC, Canada
May 1997

Jian-Jun Xu

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