

Introduction to *Mathematica*[®] with Applications

Marian Mureşan

Introduction to
Mathematica[®]
with Applications

 Springer

Marian Mureşan
Faculty of Mathematics and Computer Science
Babeş-Bolyai University
Cluj-Napoca, Romania

ISBN 978-3-319-52002-5 ISBN 978-3-319-52003-2 (eBook)
DOI 10.1007/978-3-319-52003-2

Library of Congress Control Number: 2016963590

© Springer International Publishing AG 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To my wife Viorica, always with love

Foreword

The book (written by Professor Marian Mureşan) is intended to present the *Mathematica* system in a manner in which the reader will find it easy to acquire a part of the considerable number of mathematical instruments offered by the producer, based on a large variety of examples taken from different scientific branches.

The importance of the symbolic calculus is incontestable, since nowadays the scientific calculus is not reduced to the numerical one. Modern mathematical problems, or those appearing through modeling natural phenomena, lead to such complicated symbolic expressions that we need the help of computers to process them. The fast development of the computers' technical capacity has enabled the genesis of a new domain—the symbolic (or formal) calculus—through which a genuine human–machine collaboration is achieved. The symbolic calculus systems are aimed at automating difficult calculi. They provide simple access to many sophisticated mathematical instruments, using a simple yet flexible language, that allows avoidance of writing interminable subroutines (similarly to how it happens with utilizing traditional languages).

It is mainly mathematical aspects that are covered in this book, so that the reader can understand as precisely as possible how to handle the problem. At the same time, the user is warned with regard to the type of problems which can be solved by means of the computer and the necessity of the analysis and verification of the results. Special emphasis is given to the fact that a session of *Mathematica* is typically interactive. Often there are multiple ways to approach problems, the choice of one of them being made on the spot, according to the answer of the system. The reader is carefully guided to the most suitable choice.

The book contains 11 chapters on specific aspects of the *Mathematica* language or branches of mathematics in general, such as ordinary differential equations, to which Chap. 7 is dedicated. It is worth mentioning that part of the applications presented in the book have been elaborated by the author in recent years, being the subject of certain scientific articles mentioned in the bibliography. In this book, standard linguistic constructions are used.

The book is at an incredibly high scientific level, and it is useful to many categories of users; for researchers, professors, and students in scientific fields, the knowledge of such a symbolic calculus system is nowadays indispensable. Mathematics researchers can use the programs for testing some conjectures or even for proving some results that need a big amount of calculation (in differential geometry, number theory, combinatorics, theory of functions of complex variable, numeric calculus, differential equations). The book is useful for engineers, economists, and IT specialists, who can therefore benefit from easy access to a volume of vast mathematical knowledge. I would mention that parts of the content of this book have been presented during the scientific seminars held at the Faculty of Mathematics and Computer Science, having been highly appreciated. Thanks to all of the above mentioned, I strongly recommend Prof. Marian Mureşan's book.

Cluj-Napoca, Romania
August 2016

Prof. Valeriu Anisiu

Preface

This book is the output of our work over several years. Being involved in calculus of variations and optimal control problems, we have realized that an exact calculation and a suggestive visualization are very useful, making the ideas addressed many times only in an ε - δ language clearer. Then we have chosen *Mathematica* for computations and visualizations of the ideas. Why did our option go to *Mathematica*? The answer is simple: because we had noticed the wonderful results of Prof. J. Borwein and his colleagues regarding the decimals of number π . Their approach was based on an extensive use of *Mathematica*.

Wolfram Research, located at Champaign, IL, USA, is the company which has been developing *Mathematica*.

Mathematica is continuously developing. We used *Mathematica* 10.3. It is very likely there will be newer versions with extra facilities in the future.

We have introduced notions and results in *Mathematica* in our lectures to master students at the Faculty of Mathematics and Computer Science of the Babeş-Bolyai University in Cluj-Napoca, Romania. We did the same thing with our PhD students at three summer schools organized in the framework of the grant “Center of Excellence for Applications of Mathematics” supported by DAAD, Germany. The summer schools have been organized in Struga (Macedonia, FYROM), Sarajevo (Bosnia and Herzegovina), and Cluj-Napoca (Romania).

This book is not very large, but it collects many examples. In the first part of the book, the examples are discussed in detail helping the reader to understand the reasoning in and with *Mathematica*. Later on, the reader is led to use the benefit of the **Help** and other sources freely offered by Wolfram Research. We take into account mainly the Wolfram community forum as well as the video training and conferences generously offered by Wolfram Research.

A well-motivated case for visualization in mathematics is contained in [58].

Here is the right place to express my gratitude to the following colleagues of mine from the Faculty of Mathematics and Computer Science of the Babeş-Bolyai University for their support: Anca Andreica, Valeriu Anisiu, Paul Blaga, Virginia

Niculescu, Adrian Petrușel, and Adrian Sterca. The existence and development of the MOS (Modeling, Optimization, and Simulation) Research Center of our faculty was a real help for us in the preparation of this book.

Cluj-Napoca, Romania
August 2016

Prof. Marian Mureșan

Contents

1	About <i>Mathematica</i>	1
1.1	Introduction	1
1.1.1	Warning	3
2	First Steps to <i>Mathematica</i>	5
2.1	The Introductory Techniques for Using <i>Mathematica</i>	5
2.1.1	Numbers	5
2.1.2	Bracketing in <i>Mathematica</i>	9
2.1.3	Set or SetDelayed Operator	9
2.1.4	Some Simple Steps	10
3	Basic Steps to <i>Mathematica</i>	13
3.1	Problems in Number Theory, Symbolic Manipulation, and Calculus	13
3.1.1	Problems in Number Theory	13
3.1.2	Symbolic Manipulations	17
3.1.3	Texts	19
3.2	Riemann ζ Function	20
3.3	Some Numerical Sequences	20
3.3.1	The First Sequence	20
3.3.2	The Second Sequence	21
3.3.3	The Third Sequence	22
3.4	Variables	23
3.5	Lists	24
3.5.1	Operations with Lists	24
3.5.2	Operations with Matrices	28
3.5.3	Inner and Outer Commands	31
3.5.4	Again on the Third Sequence	34
4	Sorting Algorithms	35
4.1	Introduction	35
4.2	Sorting Methods	36

- 4.2.1 Selection Sort 36
- 4.2.2 Insertion Sort..... 38
- 4.2.3 Mergesort 41
- 4.2.4 Heapsort..... 41
- 4.2.5 Quicksort 52
- 5 Functions 55**
 - 5.1 Definitions 55
 - 5.1.1 Functions with Conditions 57
 - 5.2 Differentiation and Integration 63
 - 5.2.1 Differentiation of Functions 63
 - 5.2.2 Integration of Functions 64
 - 5.3 Functions and Their Graphs 65
 - 5.4 Plane and Space Figures 68
 - 5.4.1 Plane Figures..... 68
 - 5.4.2 Functions and Graphs in 3D 85
- 6 Manipulate 93**
 - 6.1 Manipulate 93
 - 6.1.1 Circumcircle, Incircle, and the Main Points in
a Triangle 95
 - 6.1.2 Euler’s Nine Points Circle..... 97
 - 6.1.3 Frenet–Serret Trihedron of a Helix 99
 - 6.1.4 Hyperboloid of One Sheet..... 102
- 7 Ordinary Differential Equations 105**
 - 7.1 Simple Differential Equations 105
 - 7.2 Systems of Ordinary Linear Homogeneous Differential
Equations 106
 - 7.2.1 Singular Points of the Linear Homogeneous
Planar Differential Equations 110
 - 7.2.2 Multiple Equilibria 116
 - 7.3 On Two Runge–Kutta Methods of the Fourth Order 124
 - 7.3.1 An Explicit Runge–Kutta Method 124
 - 7.3.2 A Semi-explicit Runge–Kutta Method 128
- 8 Pi Formulas 133**
 - 8.1 Various Simple and Not So Simple Formulas..... 133
 - 8.1.1 Vandermonde Identity 133
 - 8.1.2 Sums Connected to Polygonal Numbers 134
 - 8.1.3 Machbin’s and Machbin-Like Formulas 134
 - 8.1.4 Gregory and Leibniz Formula..... 135
 - 8.1.5 Vardi Formula..... 136
 - 8.1.6 Abraham Sharp Formula 137
 - 8.1.7 Not So Simple Series 137
 - 8.2 Newton’s Geometric Construction 155
 - 8.3 Euler Series 155

8.4	π by Arcsin	155
8.5	π by the Golden Ratio	155
8.6	π by Integrals	157
	8.6.1 Dantzell Formula	157
	8.6.2 Lucas Formula	158
	8.6.3 Backhouse-Lucas Formula	158
8.7	BBP and Adamchik-Wagon Formulas	158
8.8	A Method for Finding AW Formulas and Proofs	159
8.9	BBP-Type Formulas for π in Powers of 2^k	160
	8.9.1 A Case	161
8.10	π Formulas by Binomial Sums	163
8.11	S. Ramanujan Series	164
8.12	R. W. Gosper Series	166
8.13	The Chudnovskys' Series	168
8.14	B. Cloitre Series	168
8.15	F. Bellard Series	169
9	Optimization of Trajectories	171
9.1	Necessary Conditions for a Mayer Optimal Control Problem	171
9.2	Zermelo's Navigation Problem	173
	9.2.1 Introduction	173
	9.2.2 A Planar Form of the Navigation Problem	174
	9.2.3 Numerical Approach to the Navigation Problem	177
9.3	Optimal Guidance for Planar Lunar Ascent	179
	9.3.1 Introduction	179
	9.3.2 The Ascent from the Lunar Surface	180
	9.3.3 The Optimal Control Approach	182
	9.3.4 Computational Issues	187
10	Miscellany in the Euclidean Plane	197
10.1	Some Planar Curves	197
	10.1.1 Conchoid of Nicomedes	197
	10.1.2 Cycloid	200
	10.1.3 Epicycloid	202
	10.1.4 Hypocycloid	207
	10.1.5 Maria Agnesi's Curve	212
	10.1.6 Cassini Ovals	213
	10.1.7 Orthoptics	214
	10.1.8 Pedals of Some Curves	220
10.2	Attractors	223
	10.2.1 Hénon Attractor	223
	10.2.2 Lorenz Attractor	225
10.3	Limit Cycles and Hopf Bifurcation	227
	10.3.1 Van Der Pol Equation	228
	10.3.2 Hopf Bifurcation	230

- 11 Miscellany in the Euclidean Space** 233
 - 11.1 Some Space Curves 233
 - 11.1.1 Viviani’s Window 233
 - 11.1.2 The Frenet–Serret Trihedron Along Viviani’s Window .. 234
 - 11.1.3 Plane Curves on a Surface 236
 - 11.2 Some Surfaces in \mathbb{R}^3 237
 - 11.2.1 Torus 237
 - 11.2.2 A Simple Tube Around Viviani’s Window 238
 - 11.2.3 Another Tube Around Viviani’s Window 238
 - 11.2.4 Helicoid, Catenoid, and Costa’s Minimal Surface 239
 - 11.3 Some Bodies 243
 - 11.3.1 The Volume of Two Bodies 243
 - 11.3.2 Reuleaux Tetrahedron 245
 - 11.4 Local Extrema of Real Functions of Several Real Variables 248
 - 11.4.1 Unconstrained Local Extrema 248
 - 11.4.2 Constrained Local Extrema 252

- References** 259

- Index** 263