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Introduction to Complex Theory of Differential Equations

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Preface

The present book is devoted to the complex theory of differential equations or, more precisely, to the theory of differential equations on complex-analytic manifolds.

The theory of differential equations on real manifolds is very well known. At the same time, the complex theory of partial differential equations is still somewhat outside the focus of interest of specialists on differential equations, which, as we believe, is completely unjust. Indeed, apart from the remarkable beauty of this theory, it can be used to solve problems in mathematics and physics in purely real situations. An example of this is given by the solution to the famous Poincaré problem on balayage inwards, and also the study of an absolutely nontrivial "mother body" problem arising in geoprospecting, as well as a number of other problems. It seems that these ideas were understood by the outstanding French mathematician Jean Leray, who together with his coauthors and pupils (L. Gårding, T. Kotake, and others) attempted to construct the theory of differential equations on complex manifolds. Unfortunately, their theory did not become as widely known, as it definitely deserved, which could be explained by the nontraditionality of the subject from the standpoint of the classical theory of partial differential equations and also by the fact that Jean Leray did not have time to solve a number of natural problems of the theory. To make the latter statement more precise, note that until recently in complex theory there was no satisfactory analogue of a transform similar to the Fourier transform in real theory, which enables solving equations with constant coefficients exactly. Moreover, one of the most important problems in complex theory, namely, the problem of describing the asymptotics of solutions of Cauchy problems, was solved by Leray only in the small, while applications required on studying singularities of the solution in the large, i.e., if we speak about Cauchy problems, far from the original manifold. This and other problems — e.g., the problem of antenna size optimization — were recently solved using complex theory. In other words, it turned out that complex theory makes it possible to solve successfully purely real problems, for instance, the balayage problem mentioned above.

It is also worth mentioning that monographs dealing with complex theory are quite difficult to understand and to some extent this probably hindered wider applications of complex theory to many important problems in physics and engineering.

The authors of the present book tried to write it so that it could be read by and be interesting to a wide range of mathematicians who may not be familiar with complicated and advanced notions in complex analysis.

Here we will share a few words about the contents of this book. The first three chapters are devoted to auxiliary material, which will be used in subsequent chapters: we describe Leray residues, and ramified integrals and their asymptotics. Chapters 4-7 are the central part of the book: here a new integral transform is introduced that allows us to obtain not only an explicit formula for the exact solution of the Cauchy problem for equations with constant coefficients, but also study singularities of these solutions. More precisely, in Chapters 4 and 5 the transform in question is introduced and its properties are discussed. Note that this transform was introduced by Sternin and Shatalov in [45,46]. In Chapters 6 and 7 we use the transform to obtain an explicit formula for the solution of the Cauchy problem and describe the singularities of the solution. Note that solutions of complex Cauchy problems always have singularities (possibly located at infinity), and therefore the function classes, in which the Cauchy problem is considered, are the classes of ramified analytic functions. Chapter 8 is devoted to studying Cauchy problems for equations with variable coefficients using Leray's uniformization method. The results obtained here are valid in the small, even though presently there is an apparatus that permits us to construct asymptotics in the large, i.e., far from the original manifold. Unfortunately, this apparatus is far more complicated technically and its exposition is beyond the scope of this book. The final Chapters 9 and 10 are devoted to applications: to the solution of Poincaré's balayage problem mentioned above as well as to an effective construction of "mother bodies."

At the end of each chapter we give bibliographic remarks on the references related to that chapter. We should mention that during the preparation of this book we widely used classical works by J. Leray and also the works by B.Yu. Sternin and V.E. Shatalov and their coauthors on complex theory of differential equations. These are briefly the contents of the book.

Acknowledgments. The results discussed in this book were delivered at a number of scientific seminars [seminars of Prof. A.S. Mishchenko and others (MSU), Acad. A.T. Fomenko (MSU), Prof. E. Schrohe (Leibniz University of Hannover, Germany)]; at international conferences in Bialowieza (Poland), Voronezh, Saint-Petersburg, Tambov (Russia) and were taught many times at various scientific centers: Independent University of Moscow, RUDN University, etc. We are grateful to all the participants of the seminars and lectures for attention and constructive criticism during the talks. The authors are also grateful to Vladimir Nazaikinskii and Pavel Sipailo.

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