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Florian Köhler-Langes

# The Electron Mass and Calcium Isotope Shifts

## High-Precision Measurements of Bound-Electron *g*-Factors of Highly Charged Ions

Doctoral Thesis accepted by
Ruperto-Carola-University of Heidelberg, Germany

Springer

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# Supervisor's Foreword

> Could anything at first sight seem more impractical than a body which is so small that its mass is an insignificant fraction of the mass of an atom of hydrogen?
>
> —Comment on the electron by J.J. Thomson, Nobel prize in physics 1906

The electron has been—ever since its discovery at the end of the nineteenth century by J.J. Thomson—subject to intense theoretical and experimental investigations. J.J. Thomson interpreted the 'cathode rays' as beams of 'corpuscles' which we now call the electron. In that time, it was a revolutionary thought to imagine a particle with very small mass as a constituent of the atom which had been considered to be an indivisible entity, a fundamental unit of matter.

Meanwhile, physicists have gained deeper understanding of the electron as an elementary particle. Nowadays, the strength of the spin magnetic moment of the electron is calculated with an accuracy of 12 digits within the relativistic theory of the electron by P.A.M. Dirac, together with contributions from quantum electrodynamics. However, within the standard model of particles, theory cannot predict the masses of the elementary constituents of matter. The particle masses rather have to be determined through experiments.

In his Ph.D. thesis, Dr. Florian Köhler-Langes determined the mass of the electron with an unprecedented accuracy of 11 digits in a precision ion trap experiment. He stored single individual ions over extended periods of time in a so-called Penning ion trap, where a combination of magnetic and electric fields is employed to confine atomic particles in a small region in space on the micrometre scale. Dr. Köhler-Langes investigated single ions such as hydrogenlike carbon ions consisting of only the atomic nucleus $^{12}C$ and one electron in the quantum mechanical ground state of the atomic shells. The experimental study of the oscillation frequencies of the trapped ions was performed under extremely good vacuum conditions of $p < 10^{-17}$ mbar, allowing for storage times of months.

In the specific case of a hydrogenlike carbon ion, Dr. Köhler-Langes measured the precession frequency of the spin motion of the electron bound in the atomic shell of the ion. At the same time, he measured the motional frequency of the carbon ion, the cyclotron frequency, orbiting in the presence of the strong magnetic field of the Penning ion trap. The combination of both frequencies yields the electron mass in atomic mass units. Hereby, the results of elaborate theoretical calculations of quantum electrodynamic effects enter as essential input parameters in the evaluation of the experimental data. The new value of the electron mass is 13 times more accurate than previous measurements and has entered the internationally recognized CODATA tables of fundamental constants. Last but not least, the experiments by Florian Köhler-Langes are a crucial test of the theory of bound-state quantum electrodynamics which accurately describes the behaviour of the electron in the presence of the extremely strong electric and magnetic fields close to an atomic nucleus.

Darmstadt, Germany
December 2016

Dr. Wolfgang Quint

# Abstract

In the context of this thesis, the electron mass has been determined in atomic mass units with a relative uncertainty of $2.8 \times 10^{-11}$, which represents a 13-fold improvement of the world literature value, the 2010 CODATA value. The underlying measurement principle combines a high-precision measurement of the Larmor-to-cyclotron frequency ratio on a single hydrogenlike carbon ion $^{12}C^{5+}$ with a very accurate *g*-factor calculation.

Furthermore, this thesis contains the first isotope shift measurement of bound-electron *g*-factors of highly charged ions. Here, the *g*-factors of the valence electrons of the lithiumlike calcium isotopes $^{40}Ca^{17+}$ and $^{48}Ca^{17+}$ have been measured with relative uncertainties of a few $10^{-10}$ constituting a so far unrivalled level of precision for lithiumlike ions. These calcium isotopes provide a unique system across the entire nuclear chart to test the pure relativistic nuclear recoil effect. The corresponding and successfully tested theoretical prediction is based on bound-state quantum electrodynamics but goes beyond the standard formalism, the so-called Furry picture, where the nucleus is considered as a classical source of the Coulomb field.

The three Larmor-to-cyclotron frequency ratios of $^{12}C^{5+}$, $^{40}Ca^{17+}$ and $^{48}Ca^{17+}$ have been determined in sequence in a non-destructive manner on single trapped ions stored in a triple Penning-trap setup. The cyclotron frequency is measured by a dedicated phase-sensitive detection technique while simultaneously probing the Larmor frequency. The spin state of the bound valence electron is determined by the continuous Stern–Gerlach effect.

In the very last part of this thesis, a new design of a highly compensated cylindrical Penning trap has been developed, which will be used in next generation's high-precision Penning-trap experiments.

# Acknowledgements

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Dear Sven Sturm, it was a fantastic fortune to work with you—such a very sympathetic and ingenious physicist—and to help you with the implementation of all your countless ideas. Your incredibly profound understanding of physics, your unique way of explaining things with so much joy and motivation, your ultra fast understanding and way of solving problems, all of that impressed me a lot. Besides all this knowledge, I would like to thank you for your sensitive, very close, daily cooperation. Without your permanent help and support such highest precision measurements would not have been possible.

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A large contribution to this thesis comes from theoretical physics. I would like to thank Jacek Zatorski, Zoltán Harmann and Christoph H. Keitel, who contributed to the determination of the electron mass. The calculations of the isotope shift and the *g*-factors of the lithiumlike calcium ions have been performed by Vladimir M. Shabaev, Dmitry A. Glazov und Andrey V. Volotka. The collaboration, the meetings and emails have been a great pleasure and quiet helpful to understand at least some part of your complicated calculations, thank you very much. In this context, I want to express my gratitude to Sergey Eliseev and the complete SHIPTRAP team for the mass measurement of $^{48}$Ca.

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# Contents

**1 Introduction** . . . . . . . . . . 1
1.1 Quantum Electrodynamics and the Standard Model of Particle Physics . . . . . . . . . . 1
1.2 Content and Structure of the Thesis . . . . . . . . . . 2
References . . . . . . . . . . 3

**2 The *g*-Factor - Exploring Atomic Structure and Fundamental Constants** . . . . . . . . . . 5
2.1 The *g*-Factor . . . . . . . . . . 6
2.2 The *g*-Factor of the Free Electron . . . . . . . . . . 7
2.3 The *g*-Factor of the Bound Electron . . . . . . . . . . 8
2.3.1 $g_{\mathrm{Dirac}}$ - Dirac Values of the Bound-Electron *g*-Factor . . . . . 10
2.3.2 $\Delta g_{\mathrm{QED}}$ - Radiative Corrections . . . . . . . . . . 12
2.3.3 $\Delta g_{\mathrm{nucl}}$ - Nuclear Corrections . . . . . . . . . . 13
2.3.4 Many-Electron Corrections . . . . . . . . . . 15
2.4 Measurement Principle of the Bound-Electron *g*-Factor . . . . . . . . . 16
2.5 Status of the Bound-Electron *g*-Factor Measurements . . . . . . . . . . 18
2.6 The Electron and Its Atomic Mass . . . . . . . . . . 19
2.6.1 Benefits of an Improved Electron Mass . . . . . . . . . . 20
2.6.2 Electron Mass Measurements in History . . . . . . . . . . 20
2.6.3 State-of-the-art Measurement Principle of the Electron Mass . . . . . . . . . . 22
2.6.4 State-of-the-art Calculations of the Bound-Electron *g*-Factor of Hydrogenlike Carbon . . . . . . . . . . 23
2.7 Testing BS-QED Beyond the Furry Picture . . . . . . . . . . 24
2.7.1 Isotope Shift Measurements . . . . . . . . . . 25
2.7.2 The Isotope Shift of the Bound-Electron *g*-Factors of Two Different Lithiumlike Calcium Isotopes, $^{40}Ca^{17+}$ and $^{48}Ca^{17+}$ . . . . . . . . . . 25

2.7.3 Theoretical Calculations of the Isotope Shift $\Delta g = g(^{40}Ca^{17+}) - g(^{48}Ca^{17+})$ . . . . . 26
2.7.4 The Atomic Masses of $^{40}Ca^{17+}$ and $^{48}Ca^{17+}$ . . . . . 26
References . . . . . 28

**3 Penning Trap Physics** . . . . . 33
3.1 Trapping Charged Particles . . . . . 33
3.1.1 The Ideal Penning Trap . . . . . 34
3.1.2 Penning Trap Imperfections . . . . . 38
3.1.3 A Real Penning Trap - Our Trap Design . . . . . 41
3.2 Induced Image Charge Detection . . . . . 43
3.2.1 The Induced Image Currents . . . . . 44
3.2.2 The Axial-Resonator - a Parallel Tank-Circuit . . . . . 45
3.2.3 Interaction Between the Trapped Ion and the Resonator . . . . . 48
3.2.4 The Image Charge Shift . . . . . 52
3.3 Excitations and Couplings of the Eigenmotions . . . . . 53
3.3.1 The Dipole Excitation . . . . . 53
3.3.2 The Quadrupole Excitation . . . . . 55
3.3.3 LC-Excitation - Electronic Feedback . . . . . 59
3.4 Various Non-thermalized Eigenfrequency Detection Methods . . . . . 61
3.4.1 Axial Peak Detection . . . . . 62
3.4.2 Phase-Sensitive Detection Methods . . . . . 64
3.5 Larmor Frequency Determination . . . . . 69
3.5.1 The Continuous Stern-Gerlach Effect . . . . . 70
3.5.2 Systematic Shifts of the Larmor Frequency . . . . . 71
References . . . . . 73

**4 Towards the Measurement of the Larmor-to-Cyclotron Frequency Ratio** . . . . . 75
4.1 Experimental Setup . . . . . 75
4.1.1 The Magnet and the Cooling Reservoirs . . . . . 76
4.1.2 Electronic Components . . . . . 78
4.1.3 The Microwave System . . . . . 80
4.1.4 The Timing System . . . . . 81
4.1.5 The Triple Penning Trap Tower . . . . . 81
4.2 Environmental Influences and Stabilization Systems . . . . . 82
4.2.1 Superconducting Compensation Coils . . . . . 83
4.2.2 Pressure Study of the Four Cooling Reservoirs . . . . . 89
4.3 Preparatory Steps and Measurements . . . . . 90
4.3.1 Creation of a Single Highly Charged Ion . . . . . 90
4.3.2 Preparation of the Precision Trap . . . . . 94
4.3.3 Preparation of the Analysis Trap . . . . . 100
4.4 The Measurement Process . . . . . 103

4.5 Line-Shape Model of the $\Gamma$-Resonance . . . . . 106
4.5.1 Sources of the Modified Cyclotron Phase Jitter . . . . . 106
4.5.2 The Combined Line-Shape Model . . . . . 111
References . . . . . 115

**5 Determination of the Atomic Mass of the Electron** . . . . . 117
5.1 Statistical $\Gamma$-Value . . . . . 117
5.2 Systematic Shifts and Uncertainties . . . . . 121
5.2.1 The Image Charge Shift . . . . . 122
5.2.2 The Line-Shape Model of the Axial Dip Signal . . . . . 122
5.2.3 Measurements of the Magnetron Frequency . . . . . 123
5.2.4 The Dipole Contribution of the Second PnA Pulse . . . . . 123
5.2.5 The Drift of the Axial Frequency . . . . . 125
5.2.6 The Image Current Shift of the Cyclotron Resonator . . . . . 125
5.2.7 Electric Field Imperfections . . . . . 126
5.2.8 Line-Shape Model of the $\Gamma$-Resonance . . . . . 126
5.2.9 Magnetic Field Imperfections . . . . . 127
5.2.10 Residual Special Relativity . . . . . 127
5.3 Consistency Checks . . . . . 128
5.4 Final Results . . . . . 130
References . . . . . 133

**6 Probing the Isotope Shift: $\Delta g = g(^{40}\mathrm{Ca}^{17+}) - g(^{48}\mathrm{Ca}^{17+})$** . . . . . 135
6.1 Spin-Flip Detection with Calcium Isotopes - the Cycle-Weight . . . 135
6.2 Statistical $\Gamma$-Values . . . . . 139
6.3 Systematic Shifts and Uncertainties . . . . . 141
6.4 Consistency Checks . . . . . 142
6.5 Final Results . . . . . 142
6.5.1 The Absolute $g$-Factors - Testing Many-Electron QED Calculations . . . . . 142
6.5.2 The $g$-Factor Difference $\Delta g = g(^{40}\mathrm{Ca}^{17+}) - g(^{48}\mathrm{Ca}^{17+})$ - Testing BS-QED beyond the Furry Picture . . . . . 144
References . . . . . 145

**7 Outlook - A New Generation of High-Precision Penning Trap** . . . . . 147
7.1 Design of a New Seven-Electrode Cylindrical Penning Trap . . . . . 148
7.1.1 Analytical Calculations of a New Trap Design . . . . . 148
7.1.2 Final Parameters of the Proton Trap . . . . . 154
7.1.3 Performance of the New Trap - Study of the Axial Frequency Shifts . . . . . 154
7.1.4 Handling the Combined Orthogonality . . . . . 156
7.1.5 Final Design of the Seven-Electrode Proton Trap . . . . . 158

7.2 Outlook . . . . . . . . . . 161
7.2.1 Bound-Electron $g$-Factor Measurements at Heavy Highly Charged Ions . . . . . . . . . . 161
7.2.2 High-Precision Measurement of the Atomic Proton Mass . . . . . . . . . . 163
References. . . . . . . . . . 164

**Curriculum Vitae** . . . . . . . . . . 167