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Mixed Convection in Fluid Superposed Porous Layers

 Springer

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Preface

Extensive research has been done in the field of natural, mixed, and forced convection in a porous layer. Several studies have investigated natural and forced convection in a system that includes a porous and a superposed fluid layer, but mixed convection has not been addressed. The present monograph is motivated to fill the gap in the literature regarding mixed convection.

We investigate mixed convective heat transfer in a long channel that is partially filled with a porous layer and has a fluid layer above it. The channel is heated on the bottom over a finite length with cross flow along the length of the channel. The two sublayers are treated as a single domain numerically, and the porosity is used as a switching parameter, causing the governing conservation equations to transition from an extended form of the Darcy-Brinkman-Forchheimer equation in the porous sublayer to the Navier-Stokes equations in the fluid sublayer. This methodology avoids the need for boundary conditions at the interface between the two domains. Dimensionless groups are varied and include the Péclet number, Rayleigh number, the porous sublayer height, Darcy number, Prandtl number, and the conductivity ratio between the solid and fluid phases. The impact of the various additional terms in the extended form of Darcy's law is also investigated.

The conductivity ratio, Darcy number, porous sublayer height, Rayleigh number, and Péclet number all have a strong effect on the overall Nusselt number, while the Prandtl number, Brinkman effect, Forchheimer effect, and convective terms have negligible effects on Nusselt numbers. A Péclet number is observed at which the Nusselt number is a minimum and is shown to be proportional to the Rayleigh-Darcy number, the product of the Rayleigh and Darcy numbers, and inversely proportional to the porous sublayer height. This Péclet number is termed the "critical Péclet number." A critical porous sublayer height ratio is also observed at which the Nusselt number is a minimum and is proportional to the Rayleigh-Darcy number and inversely proportional to the Péclet number. Streamlines capture the transition from the natural convection regime to the forced convection regime. In the transition region, flow patterns have characteristics of both. Isotherms

capture the thermal plume above the heated wall and show the influence of cross flow on the shape and character of the plume.

An experimental apparatus is designed in order to collect data over a similar range of parameters explored numerically. The numerical results show good agreement with the experimental data within the bounds of uncertainty. The experiments confirm the presence of the critical Péclet number. However, they do not show the same trends at intermediate porous layer heights. The effect of the dimensionless porous sublayer height on the Nusselt number is shown to be small from 0.5 to 1.

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Nomenclature

a	Area (m^2)
A	Heated wall area (m^2)
C_F	Forchheimer coefficient, Eqn. (2.8)
c_p	Specific heat at constant pressure (J/kg K)
d	Particle diameter (m)
Da	Darcy number, K/H^2
D_H	Channel hydraulic diameter
\mathbf{g}	Gravitational acceleration, (0, $-\mathbf{g}$) (m/s^2)
h	Heat transfer coefficient ($\text{W/m}^2\text{K}$)
H	Height (m)
k	Thermal conductivity (W/mK)
K	Permeability (m^2)
L_h	Length of heated wall (m)
Nu	Nusselt number, hH/k_m
p	Pressure (Pa)
\bar{p}	Dimensionless pressure, $pH^2/\rho_0\alpha_f^2$
Pe	Péclet number, U_0H/α_f
Pr	Prandtl number, ν_f/α_f
q	Heat transfer (W)
q'''	Energy generation (W)
Ra	Rayleigh number, $g\beta H^3(T_w - T_0)/\alpha_f\nu_f$
t	Time (s)
T	Temperature (K)
U_0	Volumetric flow/area (m/s)
\mathbf{v}	Fluid velocity, (u, v) (m/s)
$\bar{\mathbf{v}}$	Dimensionless velocity, ($u/U_0, v/U_0$)
w	Weight factor, Eqn. (1.22)
x	Horizontal coordinate (m)
y	Vertical coordinate (m)

Greek Symbols

α	Thermal diffusivity (m^2/s)
β	Isobaric coefficient of thermal expansion (K^{-1})
γ	Beavers-Joseph constant, Eqn. (1.19)
δ	Dimensionless heated wall length, L_h/H
ϕ	Porosity (—)
η	Dimensionless porous sublayer height, H_p/H
θ	Dimensionless temperature, $(T-T_0)/(T_w-T_0)$
κ	Conductivity ratio, k_s/k_f
λ	Diffusivity ratio, α_s/α_f
μ	Viscosity (Pa s)
$\tilde{\mu}$	Brinkman viscosity (Pa s)
$\hat{\mu}$	Dimensionless Brinkman viscosity $\tilde{\mu}/\mu$
ν	Kinematic viscosity (m^2/s)
τ	Dimensionless time, $t\alpha_f/H^2$
ρ	Density (kg/m^3)
σ	Heat capacity ratio, $(\rho c_p)_m/(\rho c_p)_f$
τ	Dimensionless time, $t\alpha_f/H^2$
τ_p	Dimensionless oscillatory period, Eqn. (1.21)
ω	Vorticity

Subscripts

c	Critical value
e	Effective
f	Fluid
fs	Fluid-solid interface
h	Heater
H	Total height
m	Average value
p	Porous
s	Solid
w	Wall
0	Reference