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Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

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Michele Benzi • Dario Bini • Daniel Kressner •  
Hans Munthe-Kaas • Charles Van Loan

# Exploiting Hidden Structure in Matrix Computations: Algorithms and Applications

Cetraro, Italy 2015

Michele Benzi • Valeria Simoncini

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# Preface

This volume collects the notes of the lectures delivered at the CIME Summer Course “Exploiting Hidden Structure in Matrix Computations. Algorithms and Applications,” held in Cetraro, Italy, from June 22 to June 26, 2015.

The course focused on various types of hidden and approximate structure in matrices and on their key role in various emerging applications. Matrices with special structure arise frequently in scientific and engineering problems and have long been the object of study in numerical linear algebra, as well as in matrix and operator theory. For instance, banded matrices or Toeplitz matrices fall under this category. More recently, however, researchers have begun to investigate classes of matrices for which structural properties, while present, may not be immediately obvious. Important examples include matrices with low-rank off-diagonal block structure, such as hierarchical matrices, quasi-separable matrices, and so forth. Another example is provided by matrices in which the entries follow some type of decay pattern away from the main diagonal. The structural analysis of complex networks leads to matrices that appear at first sight to be completely unstructured; a closer look, however, may reveal a great deal of latent structure, for example, in the distribution of the nonzero entries and of the eigenvalues. Knowledge of these properties can be of great importance in the design of efficient numerical methods for such problems.

In many cases, the matrix is only “approximately” structured; for instance, a matrix could be close in some norm to a matrix that has a desirable structure, such as a banded matrix or a semiseparable matrix. In such cases, it may be possible to develop solution algorithms that exploit the “near-structure” present in the matrix; for example, efficient preconditioners could be developed for the nearby structured problem and applied to the original problem. In other cases, the solution of the nearby problem, for which efficient algorithms exist, may be a sufficiently good approximation to the solution of the original problem, and the main difficulty could be detecting the “nearest” structured problem.

Another very useful kind of structure is represented by various types of symmetries. Symmetries in matrices and tensors are often linked to invariance under some group of transformations. Again, the structure, or near-structure, may not be

immediately obvious in a given problem: uncovering the hidden symmetries (and underlying transformation group) in a problem can potentially lead to very efficient algorithms.

The aim of this course was to present this increasingly important point of view to young researchers by exploiting the expertise of leading figures in this area, with different theoretical and application perspectives. The course was attended by 31 PhD students and young researchers, roughly half of them from Italy and the remaining ones from the USA, Great Britain, Germany, the Netherlands, Spain, Croatia, Czech Republic, and Finland. Many participants (about 50%) were at least partially supported by funding from CIME and the European Mathematical Society; other financial support was provided by the Università di Bologna.

Two evenings (for a total of four hours) were devoted to short 15 minute presentations by 16 of the participants on their current research activities. These contributions were of high quality and very enjoyable, contributing to the success of the meeting.

The notes collected in this volume are a faithful record of the material covered in the 28 hours of lectures comprising the course. Charles Van Loan (Cornell University, USA) discussed structured matrix computations originating from tensor analysis, in particular tensor decompositions, with low-rank and Kronecker structure playing the main role. Dario Bini (Università di Pisa, Italy) covered matrices with Toeplitz and Toeplitz-like structure, as well as rank-structured matrices, with applications to Markov modeling and queueing theory. Daniel Kressner (EPFL, Switzerland) lectured on techniques for matrices with hierarchical low-rank structures, including applications to the numerical solution of elliptic PDEs; Jonas Ballani (EPFL) also contributed to the drafting of the corresponding lecture notes contained in this volume. Michele Benzi (Emory University) discussed matrices with decay, with particular emphasis on localization in matrix functions, with applications to quantum physics and network science. The lectures of Hans Munthe-Kaas (University of Bergen, Norway) concerned the use of group-theoretic methods (including representation theory) in numerical linear algebra, with applications to the fast Fourier transform (“harmonic analysis on finite groups”) and to computational aspects of Lie groups and Lie algebras.

We hope that these lecture notes will be of use to young researchers in numerical linear algebra and scientific computing and that they will stimulate further research in this area.

*We would like to express our sincere thanks to Elvira Mascolo (CIME Director) and especially to Paolo Salani (CIME Secretary), whose unfailing support proved crucial for the success of the course. We are also grateful to the European Mathematical Society for their endorsement and financial support, and to the Dipartimento di Matematica of Università di Bologna for additional support for the lecturers.*

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# Contents

<b>Structured Matrix Problems from Tensors</b> .....	1
Charles F. Van Loan	
<b>Matrix Structures in Queuing Models</b> .....	65
Dario A. Bini	
<b>Matrices with Hierarchical Low-Rank Structures</b> .....	161
Jonas Ballani and Daniel Kressner	
<b>Localization in Matrix Computations: Theory and Applications</b> .....	211
Michele Benzi	
<b>Groups and Symmetries in Numerical Linear Algebra</b> .....	319
Hans Z. Munthe-Kaas	