

Lecture Notes in Electrical Engineering

Volume 414

Board of Series editors

Leopoldo Angrisani, Napoli, Italy
Marco Arteaga, Coyoacán, México
Samarjit Chakraborty, München, Germany
Jiming Chen, Hangzhou, P.R. China
Tan Kay Chen, Singapore, Singapore
Rüdiger Dillmann, Karlsruhe, Germany
Haibin Duan, Beijing, China
Gianluigi Ferrari, Parma, Italy
Manuel Ferre, Madrid, Spain
Sandra Hirche, München, Germany
Faryar Jabbari, Irvine, USA
Janusz Kacprzyk, Warsaw, Poland
Alaa Khamis, New Cairo City, Egypt
Torsten Kroeger, Stanford, USA
Tan Cher Ming, Singapore, Singapore
Wolfgang Minker, Ulm, Germany
Pradeep Misra, Dayton, USA
Sebastian Möller, Berlin, Germany
Subhas Mukhopadhyay, Palmerston, New Zealand
Cun-Zheng Ning, Tempe, USA
Toyoaki Nishida, Sakyo-ku, Japan
Bijaya Ketan Panigrahi, New Delhi, India
Federica Pascucci, Roma, Italy
Tariq Samad, Minneapolis, USA
Gan Woon Seng, Nanyang Avenue, Singapore
Germano Veiga, Porto, Portugal
Haitao Wu, Beijing, China
Junjie James Zhang, Charlotte, USA

About this Series

“Lecture Notes in Electrical Engineering (LNEE)” is a book series which reports the latest research and developments in Electrical Engineering, namely:

- Communication, Networks, and Information Theory
- Computer Engineering
- Signal, Image, Speech and Information Processing
- Circuits and Systems
- Bioengineering

LNEE publishes authored monographs and contributed volumes which present cutting edge research information as well as new perspectives on classical fields, while maintaining Springer’s high standards of academic excellence. Also considered for publication are lecture materials, proceedings, and other related materials of exceptionally high quality and interest. The subject matter should be original and timely, reporting the latest research and developments in all areas of electrical engineering.

The audience for the books in LNEE consists of advanced level students, researchers, and industry professionals working at the forefront of their fields. Much like Springer’s other Lecture Notes series, LNEE will be distributed through Springer’s print and electronic publishing channels.

More information about this series at <http://www.springer.com/series/7818>

Gökhan Gül

Robust and Distributed Hypothesis Testing

 Springer

Gökhan Gül
Institut für Nachrichtentechnik
Fachbereich Elektro- und
Informationstechnik (ETIT)
Technische Universität Darmstadt
Darmstadt, Germany

ISSN 1876-1100 ISSN 1876-1119 (electronic)
Lecture Notes in Electrical Engineering
ISBN 978-3-319-49285-8 ISBN 978-3-319-49286-5 (eBook)
DOI 10.1007/978-3-319-49286-5

Library of Congress Control Number: 2016956449

© Springer International Publishing AG 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To my parents

Foreword

I am delighted to introduce this book on Robust and Distributed Hypothesis Testing. The research work reported in this book was conducted by Dr. Gökhan Gül during the last five years while he was working as a research associate and studying as a Ph.D. student at the Technical University of Darmstadt. It was my great pleasure to be his thesis co-advisor and to supervise his thesis together with my dear colleague and friend Prof. Dr. Abdelhak M. Zoubir. This book comprises Dr. Gül's thesis work in which he addressed several theoretical problems that were quite challenging and the application of the results to several engineering applications, namely cognitive radio networks, forest fire detection and target image classification. The unique combination of theoretical scientific work at the university and its application to many real-world examples makes this book valuable to both theoreticians and practitioners.

This book deals with state-of-the-art problems in the area of statistical hypothesis testing in an innovative way and opens several future research directions. Although there are many books in the field of robust estimation, there are only a handful of books where robust detection has been studied. The present monograph is the first systematic, book-length exposition of robust and distributed hypothesis testing. No specific application has been targeted; therefore, in principle, the results can be applied to any engineering problem, where reliable detection of events is of interest. For example, in machine learning, new state-of-the-art supervised classifiers that have strong robustness properties can be designed. It also provides an alternate way to mitigate the model mismatch problem often encountered in image processing problems. Moreover, the material in this book is self-contained and does not make use of the concepts and proofs developed by others in the literature. Hence, this book is very easy to follow.

In statistics, robust hypothesis testing refers to the reliable detection of events in the presence of uncertainties regarding the assumed model. The uncertainties may be in the form of outliers, which are the observations that do not follow the majority of the data, or in the form of modeling errors which are caused by imperfect calibration, changes in the environment, as well as the presence of interfering

signals. Following Chap. 2, where background information is presented, Chaps. 3 and 4 are devoted to the design of minimax robust tests considering two different types of uncertainties. In many applications, such as radar, sonar and wireless sensor networks, multiple sensors are available. Furthermore, it may be of interest to give decisions sequentially, without waiting for a certain block of observations. This leads to the material presented in Chap. 5, where the designed robust statistical tests have been extended to sequential hypothesis testing, and in Chap. 6, where a thorough design of minimax robust distributed hypothesis testing has been carried out. It is worth mentioning that the proposed theoretical models are applicable to robust estimation problems as well. Finally, before the conclusions, some novel theoretical bounds in distributed detection have been derived. These bounds will be of great interest to researchers and are obtained for the first time in the literature. I believe that the reader will benefit immensely from reading this monograph which breaks new grounds in the area of statistical decision theory and robust statistics.

Pramod K. Varshney
Distinguished Professor at Syracuse University
Syracuse, NY, USA

Acknowledgments

It gives me great pleasure to express my sincere gratitude to all people who have supported and helped me in the preparation of this book.

My deepest gratitude is to my advisor, Prof. Dr.-Ing Abdelhak M. Zoubir, for his patient guidance, both technical and spiritual continuous support, enthusiastic encouragement, and useful critiques. He has been a role model for me from whom I learned many things. It was possible only with his help to increase my ability to think and correct my mistakes. He has provided me enough freedom during my research, and he has always been nice to me. I would like to thank him especially for giving me the chance to pursue my Ph.D. study under his supervision with a very interesting and challenging topic.

I would like to also thank Prof. Dr. Pramod K. Varshney for being my co-supervisor as well as Prof. Dr. Marius Pesavento, Prof. Dr. Andy Schürr, and Prof. Dr. Thomas Weiland who acted as the examiners and the chair in the Ph.D. committee.

Many thanks go to the current and former colleagues at the Signal Processing Group at TU Darmstadt. I would like to especially thank Michael Fauß for his valuable feedback and our fruitful discussions. It was a great pleasure to work with brilliant researchers in a very international atmosphere. I wish to extend my thanks to our secretaries Renate Koschella and Christina Cramer and our system administrator Hauke Fath. All helped me very much to take care of all non-scientific works.

I would like to express my very great appreciation to Dr. Roy Howard for his valuable and constructive suggestions. His willingness to give his time so generously has been very much appreciated. My special thanks go to Prof. Dr. Fatih Kurugöllü for proofreading my dissertation, Prof. Dr. Didier Piau for his assistance, and Prof. Dr. Bernard C. Levy for kindly answering all my questions about his publications.

I gratefully acknowledge the funding received by the Cocoon project and the Deutscher Akademischer Austausch Dienst (DAAD) for covering my conference costs in Lisbon, Portugal, and Gold Coast, Australia. I had amazing times with my

colleagues in Australia and Portugal. Thank you very much Nevine Dimitri, Michael Muma, Stephan Vlaski, and Christian Weiss for sharing a beautiful atmosphere with me.

Most importantly, I wish to thank my parents Pakize and Kenan Gül for their unconditional love, support and encouragement throughout my study. Last but not least, I would like to thank the rest of my family, especially my grandmother Hanife Cebeci.

Contents

1	Introduction	1
1.1	Motivation	1
1.2	Related Work	7
1.3	Contributions	8
1.3.1	Publications	9
1.4	Book Overview	10
	References	11
2	Background	15
2.1	Introduction	15
2.2	Robust Detection	15
2.2.1	Minimax Hypothesis Testing	16
2.2.2	Robust Hypothesis Testing	20
2.3	Decentralized Detection	24
2.4	Conclusions	25
	References	25
3	Robust Hypothesis Testing with a Single Distance	27
3.1	Introduction	27
3.2	Huber's Minimax Robust Hypothesis Test	27
3.2.1	LFDs and the Existence of Saddle Value	28
3.2.2	Distributions of the Log-Likelihood Ratios of LFDs	29
3.2.3	Limiting Robustness Parameters	31
3.2.4	Limiting Test	32
3.3	Minimax Robust Hypothesis Testing with KL-Divergence	32
3.3.1	Saddle Value Specification	33
3.3.2	Problem Definition	34
3.3.3	Derivation of LFDs and the Robust Decision Rule	34
3.3.4	Distribution of the Log-Likelihood Ratios of LFDs	38
3.3.5	Monotonicity of KL-Divergence	39
3.3.6	Symmetric Density Functions	41
3.3.7	Limiting Robustness Parameters	41
3.3.8	Limiting Test	44

- 3.4 Other Distances 44
 - 3.4.1 The χ^2 - and squared Hellinger distance. 45
 - 3.4.2 Symmetrized χ^2 -distance 45
 - 3.4.3 Symmetrized KL-divergence 45
- 3.5 Asymptotically Robust Hypothesis Test 46
 - 3.5.1 Limiting Test 47
- 3.6 Simulations 47
 - 3.6.1 Theoretical Examples 47
- 3.7 Conclusions 50
- References. 51
- 4 Robust Hypothesis Testing with Multiple Distances 53**
 - 4.1 Introduction 53
 - 4.2 Huber’s Generalized Minimax Robust Hypothesis Test 54
 - 4.2.1 Distributions of the Log-Likelihood Ratios of LFDs. 55
 - 4.3 Robust Hypothesis Testing with α -Divergence 55
 - 4.3.1 Saddle Value Specification 56
 - 4.3.2 Problem Definition 57
 - 4.3.3 Derivation of LFDs and the Robust Decision Rule. 57
 - 4.3.4 Distributions of the Log-Likelihood Ratios of LFDs. 62
 - 4.3.5 Simplified Model with Additional Constraints. 63
 - 4.3.6 Limiting Robustness Parameters 64
 - 4.3.7 Limiting Test 66
 - 4.4 Robust Hypothesis Testing with Composite Distances 66
 - 4.4.1 Composite Uncertainty Model. 67
 - 4.4.2 Existence of Least Favorable Distributions 67
 - 4.4.3 Two Examples of the Composite Test. 68
 - 4.5 Simulations 70
 - 4.5.1 Theoretical Examples 70
 - 4.5.2 Signal Processing Example: Spectrum Sensing 76
 - 4.6 Conclusions 78
 - References. 79
- 5 Robust Hypothesis Testing with Repeated Observations 81**
 - 5.1 Introduction 81
 - 5.2 Robust Fixed Sample Size Tests. 81
 - 5.2.1 Fixed Sample Size (h)-Test. 82
 - 5.2.2 Fixed Sample Size $(m)_\alpha$ -Test 82
 - 5.2.3 Fixed Sample Size (c)-Test 83
 - 5.2.4 Asymptotic Performance Analysis. 83
 - 5.3 Robust Sequential Probability Ratio Tests 85
 - 5.3.1 Sequential (h)-Test 87
 - 5.3.2 Sequential $(m)_\alpha$ - and (c)-Test 88
 - 5.3.3 Sequential (a)-Test 88
 - 5.4 An Extension of the Composite Model
to Robust Estimation Problems. 88

5.5	Simulations	89
5.5.1	Theoretical Examples	89
5.5.2	Signal Processing Example: Target Image Classification	93
5.6	Conclusions	96
	References.	97
6	Robust Decentralized Hypothesis Testing	99
6.1	Introduction	99
6.2	System Specification and Problem Definition	100
6.3	General Solutions to Robust Decentralized Detection Problem.	101
6.4	Specific Examples.	105
6.4.1	Huber’s Extended Uncertainty Class	105
6.4.2	Uncertainty Classes Based on α -Divergence	105
6.4.3	Composite Uncertainty Classes	106
6.5	Generalizations	106
6.5.1	Neyman–Pearson Formulation.	106
6.5.2	Repeated Observations and Centralized Detection.	107
6.5.3	Different Network Topologies.	107
6.6	Simulations.	108
6.6.1	Signal Processing Example: Forest Fire Detection	108
6.7	Conclusions	110
	References.	111
7	Minimax Decentralized Hypothesis Testing	113
7.1	Introduction	113
7.2	Constraints in the Design of Minimax DDN-WoF	114
7.2.1	Constraints on the System Design.	114
7.2.2	Constraints on the Achievable Performance	117
7.3	The Maximum Loss Due to Minimax Decision Making in DDN-WoF	117
7.3.1	Single Sensor Case	119
7.3.2	Multiple Sensor Case	121
7.4	The Maximum Loss Between Minimax DDN-WoF and DDN-WF.	123
7.4.1	Derivation of the Maximum Performance Loss.	124
7.4.2	Generalizations	128
7.5	Conclusions	129
	References.	129
8	Conclusions and Outlook	131
8.1	Conclusions	131
8.2	Outlook	132
	Erratum to: Robust Hypothesis Testing with a Single Distance	E1
	Appendix A	135

About the Author



Gökhan Gül (S'09) was born in Amasya, Turkey, in 1982. He received the B.Sc. degree (with first class honor) in electronic engineering from Uludağ University, Bursa, Turkey, in 2005, and the M.Sc. degree in Digital Communications from Christian Albrechts University, Kiel, Germany, in 2009. He wrote his master thesis and worked as a research assistant in Fraunhofer Institute (IIS), Erlangen (2008–2009). From 2010 to 2011, he was a visiting researcher at Queen's University, Belfast, UK. Gökhan Gül is the recipient of the second-degree award of the first international competition on steganalysis (HUGO) in 2011. This same year, he joined

Signal Processing Group, Technische Universität Darmstadt, Germany, as a research associate, and in 2015, Gökhan Gül received his Dr.-Ing. degree (with distinction) from the same institute. His research interests lie in detection and estimation theory, statistics, probability theory, and general topology.

Acronyms

ATR	Automatic Target Recognition
AUC	Area Under the Curve
BPSK	Binary Phase-Shift Keying
c.d.f.	Cumulative Distribution Function
DC	Direct Current
DDN-WF	Distributed Detection Network With a Fusion Center
DDN-WoF	Distributed Detection Network Without a Fusion Center
FKG	Fortuin–Kasteleyn–Ginibre
i.i.d.	Independent and Identically Distributed
KKT	Karush–Kuhn–Tucker
KL	Kullback–Leibler
LFD	Least Favorable Distribution
MSTAR	Moving and Stationary Target Acquisition and Recognition
NP	Non-deterministic Polynomial Time
PBPO	Person-By-Person Optimum
r.v.	Random Variable
ROC	Receiver Operating Characteristic
SAR	Synthetic Aperture Sonar
SNR	Signal-to-Noise Ratio
SPRT	Sequential Probability Ratio Test

Symbols

$\mathbf{1}(\cdot)$	Indicator function
$\mathbf{1}$	Vector of ones
t_l	Lower threshold for the sequential test
t_u	Upper threshold for the sequential test
ϕ	Non-randomized decision rule
ϕ_r	Non-randomized robust decision rule
ϕ_0	Non-randomized optimum decision rule
δ	Randomized decision rule
δ_x	Dirac delta function
ϵ	A small number
μ_j, λ_j	Lagrangian multipliers
Ω	Sample space
\mathcal{A}	Sigma algebra on the sample space Ω
D	Distance between probability measures
\mathcal{H}_0	Null hypothesis
\mathcal{H}_1	Alternative hypothesis
χ^2	χ^2 distance
μ	A measure on a measurable space
R	Risk function
f_0	Nominal probability density function under the null hypothesis for distance D
f_1	Nominal probability density function under the alternative hypothesis for distance D
F_0	Nominal probability distribution function under the null hypothesis for distance D
F_1	Nominal probability distribution function under the alternative hypothesis for distance D
g_0	Actual probability density function under the null hypothesis for distance D

g_1	Actual probability density function under the alternative hypothesis for distance D
G_0	Actual probability distribution function under the null hypothesis for distance D
G_1	Actual probability distribution function under the alternative hypothesis for distance D
\hat{g}_0	Least favorable probability density function under the null hypothesis for distance D
\hat{g}_1	Least favorable probability density function under the alternative hypothesis for distance D
\hat{G}_0	Least favorable probability distribution function under the null hypothesis for distance D
\hat{G}_1	Least favorable probability distribution function under the alternative hypothesis for distance D
\mathcal{G}_0	Uncertainty set under the null hypothesis for distance D
\mathcal{G}_1	Uncertainty set under the alternative hypothesis for distance D
ε_0	Robustness parameter under the null hypothesis with respect to distance D
ε_1	Robustness parameter under the alternative hypothesis with respect to distance D
l_l	Lower threshold of the minimax robust test with respect to distance D
l_u	Upper threshold of the minimax robust test with respect to distance D
p_0	Nominal probability density function under the null hypothesis for ϵ -contamination model
p_1	Nominal probability density function under the alternative hypothesis for ϵ -contamination model
P_0	Nominal probability distribution function under the null hypothesis for ϵ -contamination model
P_1	Nominal probability distribution function under the alternative hypothesis for ϵ -contamination model
q_0	Actual probability density function under the null hypothesis for ϵ -contamination model
q_1	Actual probability density function under the alternative hypothesis for ϵ -contamination model
Q_0	Actual probability distribution function under the null hypothesis for ϵ -contamination model
Q_1	Actual probability distribution function under the alternative hypothesis for ϵ -contamination model
\hat{q}_0	Least favorable probability density function under the null hypothesis for ϵ -contamination model
\hat{q}_1	Least favorable probability density function under the alternative hypothesis for ϵ -contamination model
\hat{Q}_0	Least favorable probability distribution function under the null hypothesis for ϵ -contamination model

\hat{Q}_1	Least favorable probability distribution function under the alternative hypothesis for ϵ -contamination model
\mathcal{P}_0	Uncertainty set under the null hypothesis for Huber's test
\mathcal{P}_1	Uncertainty set under the alternative hypothesis for Huber's test
ϵ_0	Robustness parameter under the null hypothesis with respect to ϵ -contamination model
ϵ_1	Robustness parameter under the alternative hypothesis with respect to ϵ -contamination model
b	Maximum bias function
c_l	Lower threshold of Huber's clipped likelihood ratio test
c_u	Upper threshold of Huber's clipped likelihood ratio test
n	Total number of samples
K	Total number of sensors
τ	Stopping time of stochastic process S_n
S_n	A stochastic process
h_0^K	Fusion function for K sensors
γ^K	Binary fusion rule for K sensors
y	An observation
W	Lambert W -function
Δ	Set of decision rules
π_j	A priori probability
ρ	Ratio of the a priori probabilities π_0/π_1
C_{ij}	Cost of making a decision i when hypothesis j is true
P_F	False alarm probability
P_M	Miss detection probability
P_E	Overall error probability
T	Test statistic
\mathcal{L}	Loss function