New Advances on Chaotic Intermittency and its Applications

Sergio Elaskar • Ezequiel del Río

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Sergio Elaskar
National University of Cordoba
Ciudad de Córdoba
Córdoba, Argentina

Ezequiel del Río
Polytechnic University of Madrid
Madrid, Madrid, Spain

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To my wife, Mila, and my daughter, Alba
(EdR)

To my family, María Inés, Silvina and Daniel
(SE)

Preface

From the pioneer works of H. Poincaré to present days, the nonlinear science has developed tremendously. In particular, chaos theory presents nowadays a number of fields of investigation. One of such fields is the so-called routes to chaos, where an interesting one is the chaotic intermittency, because this phenomenon has been observed in many different fields. The intermittency theory coming from the early 1980s was based on a strong hypothesis on the reinjection probability density function (RPD).

From early times, “pathology cases” were found, that is, systems showing chaotic intermittency with statistic properties not fully explained basing on the classical theory. These cases demand a broader intermittency theory looking for a new and more general RPD function.

By means of the Poincaré map, many continuous systems can be investigated by one-dimensional maps. In this book, new methodologies to investigate chaotic intermittency in one-dimensional maps are presented. A new general methodology to evaluate the RPD is developed. The core of this formulation is a new function, called $M(x)$, which is very useful to calculate the RPD function, even for a small number of numerical or experimental data. The $M(x)$ function is defined by means of integrals; hence the influence on the statistical fluctuations in the data series is reduced.

As a result, a more general form for the RPD is found. By including the new RPD in the classical mathematical formulation of chaotic intermittency, new results have been obtained. For instance, the characteristic exponent, traditionally used to characterize the intermittency type, is now a function depending on the whole map, not just on the local map. In this new framework, the classical theory is recovered as a particular case. Even more, the pathology cases are included in a natural way in the new theory. Also, we present a new analytical approach to obtain the RPD from the mathematical expression of the map.

In this new framework, the noise effect on the system is evaluated by means of the analytical derivation of the noisy RPD (NRPD). This is an important difference with respect to the classical approach based on the Fokker–Planck equation or renormalization group theory, where the noise effect was usually considered just on the local Poincaré map.

Finally, a new scheme to evaluate the RPD function using the Perron–Frobenius operator is developed. We present along the book examples of applications of these methodologies used to evaluate the RPD. In every case, they have shown good agreement with numerical computations.

Ciudad de Córdoba, Argentina — Sergio Elaskar
Madrid, Spain — Ezequiel del Río
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Contents

Acronyms

CDF	Channel distribution function
DHS	d-Dimensional diagonal hypersurface
DNLS	Derivative nonlinear Schrodinger equation
EEG	Electroencephalography
ELL	Electrosensory lateral line lobe
FPE	Backward Fokker-Planck equation
HH	Hodgkin–Huxley neuron model
HV	Horizontal visibility
HVg	Horizontal visibility graph
ISI	Inter-spike intervals: time between two consecutive spikes
IPI	Inter-peak intervals: time between two consecutive peaks
LBR	Lower boundary of reinjection: limit value for the reinjection from the chaotic region into the laminar one
MFPT	Mean first-passage time
NDP	Non-differentiable points
NRPD	Noisy reinjection probability density: the RPD modified by the noise effect
op-amp	Operational amplifier
RC	Resistor-capacitor
RHS	Right-hand side
RGT	Renormalization group theory
RPD	Reinjection probability density from the chaotic region into the laminar one

Nomenclature

a	Real number
b	Normalization constant
c	Upper limit of the laminar interval
D	Compact manifold
$F(x), G(x)$	One-dimensional maps
h	Integer number
j	Integer number
k	Real number
l	Laminar length
$\bar{l}$	Average laminar length
m	Slope of the function $M(x)$
n	Integer number
$P\circ$	The Perron–Frobenius operator
$SF(x)$	The Schwartzian derivative
t	Time
$\hat{x}$	Lower boundary of reinjection
$\tilde{x}$	Lower boundary of return
$\mathbf{f}$	Vector field for continuous systems
$\mathbf{F}$	Vector field for discrete systems
$\mathbf{H}$	Vector containing all control parameters of the system
$\mathbf{n}$	Unit normal vector to the hypersurface
$\mathbf{x}$	State vector
α	Exponent of the power law modelling RPD
β	Characteristic exponent
Δ	Real number
ε	Control parameter
$\rho(x)$	Density
$\mu(x)$	Measure

$\phi(x)$	Noiseless reinjection probability density function
$\Phi(x)$	Noisy reinjection probability density function
$\psi(l)$	Probability density of the laminar lengths
$\Theta(x)$	Heaviside step function
Σ	Hypersurface