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Algebraic Geometry and Number Theory

Summer School, Galatasaray University, Istanbul, 2014

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Preface

The CIMPA summer school "Algebraic Geometry and Number Theory" (AGNT) was held at Galatasaray University, Istanbul, between 2 and 13 June 2014. Subjects ranging from Arakelov geometry and Iwasawa theory to classical projective geometry and birational geometry were covered. All these active research domains were presented by leading experts in their respective fields. The main aim of the lectures was to introduce these contemporary research topics to graduate students. Accordingly the lectures have incorporated the main ideas and techniques together with motivating examples and guiding problems in this variety of subjects.

Each individual lecturer provided a detailed outline of the lecture together with a list of prerequisites. In addition, lecturers have pre-organized the lectures so as to use the two-week time in most efficient way.

The young participants have shown a considerable interest to the summer school. We had to regretfully decline more than half of the applications to our school. Those who participated have been actively involved in the lectures and helped providing a lively atmosphere throughout the school. The final three days of the summer school afternoon sessions were reserved for research talks. The talks were mostly delivered by the young participants.

We are thankful to Galatasaray University not only for the financial, but also for the logistic support. We would also like to thank CIMPA and TÜBİTAK for their financial support.

Our gratitude goes also to the members of the scientific committee (Olivier Debarre (École Normale Supérieure, Paris, France), Fouad El Zein (IMJ, Paris, France), Monique Lejeune (Université de Versailles, France), Kamal Khuri-Makdisi (LSMS, Beirut, Lebanon), Rahim Zaare-Nahandi (University of Tehran, Iran), Chris Peters (Université de Grenoble, Institut Fourier, France), Christophe Soulé (IHÉS, France), Loring W. Tu (Tufts University, Medford, USA), A. Muhammed Uludağ (Galatasaray University, Turkey)) and those of the organizing committee (Hakan Ayral (Galatasaray University, Turkey), Merve Durmuş (Yeditepe University, Turkey), Hussein Mourtada (Université de Paris VII, France), İrem Portakal (Galatasaray University, Turkey), İsmail Sağlam (Koç University, Turkey), Celal Cem Sarıoğlu (Dokuz Eylül University, Turkey), Ayberk Zeytin (Galatasaray University, Turkey)) for their efforts.

The last named editor was supported by TÜBİTAK grants 113R017 and 114R073 during the summer school and during the preparation of this volume.

We now offer a glance of the program:

Kazım Büyükboduk (Koç University, Turkey):
Arithmetic of Abelian varieties and Iwasawa theory

İzzet Coşkun (University of Illinois, Chicago, USA):
Birational geometry of moduli spaces

Olivier Debarre (École Normale Supérieure, Paris, France):
On the geometry of subvarieties of low degree in the complex projective space

Sabir M. Gusein-Zade (Moscow State University, Moscow, Russia):
Singular points of complex hypersurfaces

Rahim Zaare-Nahandi (University of Tehran, Iran):
A brief introduction to computational commutative algebra

Chris Peters (Université de Grenoble, Institut Fourier, France):
Lectures on motivic aspects of Hodge theory

Christophe Soulé (IHÉS, France) and **Gerard Freixas i Montplet** (Institut de Mathématiques de Jussieu, Paris, France):
Arakelov theory

Loring W. Tu (Tufts University, Medford, USA):
Sheaf cohomology

Sinan Ünver (Koç University, İstanbul, Turkey):
Arakelov geometry on arithmetic surfaces

The following is an outline of the individual chapters.

Büyükboduk, in his chapter, gives an overview of Iwasawa theory and considers the following issues:

- Kummer's work on cyclotomic fields and Fermat's last theorem: Kummer congruences for the special values of the Riemann zeta-function,
- Iwasawa theory of cyclotomic fields,
- Iwasawa theory of other Galois representations (and motives), after Greenberg and Mazur,
- Iwasawa theory of Galois deformations and applications (to the Taniyama-Shimura conjecture and Fermat's Last Theorem, the Sato-Tate, Birch and Swinnerton-Dyer (and its p-adic variants), Artin conjectures and the Langlands' program),

Coşkun's contribution is on birational geometry of moduli spaces. After introducing the basic objects and techniques used in birational geometry and Mori program, he demonstrates the theory through simple examples. He then studies the birational geometry of the Hilbert scheme of points on $\mathbf{P}^2$. He also discusses

other moduli spaces and gives a guide to the literature in this subject. The text contains many exercises.

Debarre gives a modern treatment of some problems of classical algebraic geometry. In his text entitled "On the geometry of hypersurfaces of low degrees in the projective space", he considers the following issues:

- Projective spaces and Grassmannians, Schubert calculus,
- Projective lines on a hypersurfaces,
- Rationality of smooth cubic surfaces over algebraically closed fields. Segre non-rationality criterion in the case of non-algebraically closed fields,
- The intermediate Jacobian, the Albanese variety, principally polarized abelian varieties and theta divisors, Abel-Jacobi maps, conic bundles, and Prym varieties. The Clemens-Griffiths proof of the fact that cubic hypersurfaces of dimension 3 over the complex numbers are not rational,
- Cubic fourfolds.

Freixas' contribution is a friendly introduction to Arakelov geometry. He begins with the proof of the finiteness of class groups following Minkowski's approach but using the modern language of algebraic geometry. In this way, he motivates the arithmetic Riemann-Roch theorem, whose formulation is the main goal of the lectures. He then presents:

- The Grothendieck-Riemann-Roch theorem,
- The arithmetic Chow groups and characteristic classes,
- The arithmetic Riemann-Roch theorem,
- Some applications of the arithmetic Riemann-Roch theorem.

Tu, in his chapter, makes use of the localization formula in equivariant cohomology to give a systematic method to compute the Gysin homomorphism in the ordinary cohomology of a fiber bundle. Then he recovers as examples classical push-forward formulas for generalized flag bundles.

The contribution by Wojtkowiak is around the idea of obtaining information regarding certain arithmetic data by p-adic interpolation. The main aim is to interpolate special values of the Dirichlet L-functions $L(s, \psi)$; where ψ is a finite order character on $\mathbf{Z}_p^\times$. This is achieved by constructing a very natural sequence of measures. Besides constructing the measures, and relating the very first of these measures to p-adic Dirichlet L-functions – which are the main theorems of the paper – the author also proves various other facts about the measures.

In his chapter, Zeytin studies a very classical topic: integral binary quadratic forms. A new approach to this subject using certain bipartite ribbon graphs called çarks is given. The central result of the paper is the one to one correspondence between integral points of an affine hypersurface admitting a certain group action and indefinite binary quadratic forms of fixed discriminant. The author also studies these hypersurfaces along the lines of Kobayashi hyperbolicity. More precisely, by a

result of Demailly these hypersurfaces are Kobayashi hyperbolic and have finitely many rational points as the class group is finite. These results are in accordance with a conjecture of Lang relating arithmetic to hyperbolicity.

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