Dimensional Analysis Beyond the Pi Theorem

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To one of my best friends Bill Kemp, his wife Linda, and daughter Jennifer

About the Author

Dr. Bahman Zohuri currently works for Galaxy Advanced Engineering, Inc., a consulting firm that he started in 1991 when he left both the semiconductor and defense industries after many years working as a chief scientist. After graduating from the University of Illinois in the field of physics and applied mathematics, he then went to the University of New Mexico, where he studied nuclear engineering and mechanical engineering. He joined Westinghouse Electric Corporation, where he performed thermal hydraulic analysis and studied natural circulation in an inherent shutdown and heat removal system (ISHRS) in the core of a liquid metal fast breeder reactor (LMFBR) as a secondary fully inherent shutdown system for secondary loop heat exchange. All these designs were used in nuclear safety and reliability engineering for a self-actuated shutdown system. He designed a mercury heat pipe and electromagnetic pumps for large pool concepts of an LMFBR for heat rejection purposes for this reactor around 1978, when he received a patent for it. He was subsequently transferred to the defense division of Westinghouse, where he oversaw dynamic analysis and methods of launching and controlling MX missiles from canisters. The results were applied to MX launch seal performance and muzzle blast phenomena analysis (i.e., missile vibration and hydrodynamic shock formation). Dr. Zohuri was also involved in analytical calculations and computations in the study of nonlinear ion waves in rarefying plasma. The results were applied to the propagation of so-called soliton waves and the resulting charge collector traces in the rarefaction characterization of the corona of laser-irradiated target pellets. As part of his graduate research work at Argonne National Laboratory, he performed computations and programming of multi-exchange integrals in surface physics and solid-state physics. He earned various patents in areas such as diffusion processes and diffusion furnace design while working as a senior process engineer at various semiconductor companies, such as Intel Corp., Varian Medical Systems, and National Semiconductor Corporation. He later joined Lockheed Martin Missile and Aerospace Corporation as senior chief scientist and oversaw research and development (R&D) and the study of the vulnerability, survivability, and both radiation and laser hardening of different components of the Strategic Defense Initiative, known as Star Wars.

This included payloads (i.e., IR sensor) for the Defense Support Program, the Boost Surveillance and Tracking System, and Space Surveillance and Tracking Satellite against laser and nuclear threats. While at Lockheed Martin, he also performed analyses of laser beam characteristics and nuclear radiation interactions with materials, transient radiation effects in electronics, electromagnetic pulses, system-generated electromagnetic pulses, single-event upset, blast, and thermomechanical, hardness assurance, maintenance, and device technology.

He spent several years as a consultant at Galaxy Advanced Engineering serving Sandia National Laboratories, where he supported the development of operational hazard assessments for the Air Force Safety Center in collaboration with other researchers and third parties. Ultimately, the results were included in Air Force Instructions issued specifically for directed energy weapons operational safety. He completed the first version of a comprehensive library of detailed laser tools for airborne lasers, advanced tactical lasers, tactical high-energy lasers, and mobile/ tactical high-energy lasers.

He also oversaw SDI computer programs, in connection with Battle Management C^3I and artificial intelligence, and autonomous systems. He is the author of several publications and holds several patents, such as for a laser-activated radioactive decay and results of a through-bulkhead initiator. He has published the following works: *Heat Pipe Design and Technology: A Practical Approach* (CRC Press); *Dimensional Analysis and Self-Similarity Methods for Engineers and Scientists* (Springer); *High Energy Laser (HEL): Tomorrow's Weapon in Directed Energy Weapons Volume I* (Trafford Publishing Company); and recently the book on the subjects directed energy weapons and physics of high-energy laser with Springer. He has other books with Springer Publishing Company: *Thermodynamics in Nuclear Power Plant Systems* and *Thermal-Hydraulic Analysis of Nuclear Reactors*.

Preface

In physics and science, dimensional analysis is a tool to find or check relations among physical quantities by using their dimensions. The dimension of a physical quantity is the combination of the basic physical dimensions (usually mass, length, time, electric charge, and temperature) which describe it. For example, speed has the dimension length/time and may be measured in meters per second, miles per hour, or other units. Dimensional analysis is necessary because a physical law must be independent of the units used to measure the physical variables in order to be general for all cases.

Dimensional analysis is routinely used to check the plausibility of derived equations and computations as well as forming reasonable hypotheses about complex physical situations that can be tested by experiment or by more developed theories of the phenomena, which allow categorizing the types of physical quantities. In this case, units are based on their relations or dependence on other units or dimensions, if any.

Isaac Newton (1686) who referred to it as the "Great Principle of Similitude" understood the basic principle of dimensional analysis. The nineteenth-century French mathematician Joseph Fourier made important contributions based on the idea that physical laws like $F = MA$ should be independent of the units employed to measure the physical variables. This led to the conclusion that meaningful laws must be homogeneous equations in their various units of measurement, a result that was eventually formalized by Edgar Buckingham with the π (pi) theorem. This theorem describes how every physically meaningful equation involving n variables can be equivalently rewritten as an equation of n–m dimensionless parameters, where m is the number of fundamental dimensions used. Furthermore, and most importantly, it provides a method for computing these dimensionless parameters from the given variables.

A dimensional equation can have the dimensions reduced or eliminated through nondimensionalization, which begins with dimensional analysis and involves scaling quantities by characteristic units of a system or natural units of nature.

The similarity method is one of the standard methods for obtaining exact solutions of partial differential equations (PDEs), in particular nonlinear forms. The number of independent variables in a PDE is reduced one by one to make use of appropriate combinations of the original independent variables as new independent variables, called "similarity variables."

In some cases, dimensional analysis does not provide an adequate approach to establish a solution of a certain eigenvalue problem in nonlinear form which gives rise to the need to discuss similarity method as another approach. In particular, simple cases deal with reduction of a partial differential equation to an ordinary differential equation in an ordinary way that we have learned in any classical text of the same type. In more scenarios that are complex, dealing with boundary value problem for a system of ordinary equations with conditions at different ends of an infinite interval requires to construct a self-similar solution that is a more efficient way of solving such complex boundary value problem for the system of ordinary equations directly. In a specific instance, the passage of the solution into a self-similar intermediate asymptotic prevents a return to the partial differential equations; indeed, in many cases, the self-similarity of intermediate asymptotic can be established and the form of self-similar intermediate asymptotic obtained from dimensional considerations.

For the subject of this book *Dimensional Analysis Beyond the Pi Theorem*, we are looking beyond just the simple pi theorem. Although the dimensional analysis and physical similarity are well-understood subjects, the general concepts of dynamical similarity are explained in this book. Our exposition is essentially different from those available in the literature, although it follows in its general ideas that are known as the pi theorem and there are many excellent books by the different authors that are published, which one can refer to. However, dimensional analysis goes way beyond the pi theorem, which is also known as Buckingham's pi theorem. Many techniques via self-similar solutions can bound these solutions to problems that seem to be intractable.

The human partner in the interaction of a man and a computer often turns out to be the weak spot in the relationship. The problem of formulating rules and extracting ideas from vast masses of computational or experimental results remains a matter for our brains, our minds. This problem is closely connected with the recognition of patterns. The word "obvious" has two meanings, not only something easily and clearly understood but also something immediately evident to our eyes. The identification of forms and the search for invariant relations constitute the foundation of pattern recognition; thus, we identify the similarity of large and small triangles.

A time-developing phenomenon is called self-similarity if the spatial distributions of its properties at various different moments of time can be obtained from one another by a similarity transformation, and the fact that we identify one of the independent variables of dimension with time is nothing new from the subject of dimensional analysis point of view. However, this is where the boundary of dimensional analysis goes beyond the pi theorem and steps into a new arena that is known as self-similarity, which has always represented progress for researchers.

In recent years, there has been a surge of interest in self-similar solutions of the first and second kind. Such solutions are not newly discovered; they had been identified and in fact so named by Zel'dovich, a famous Russian mathematician, in 1956, in the context of a variety of problems, such as shock waves in gas dynamics and filtration through elastoplastic materials.

Self-similarity has simplified computations and the representation of the properties of phenomena under investigation. It handles experimental data, reduces what would be a random cloud of empirical points to lie on a single curve or surface, and constructs procedure that is known to us as self-similar, wherein variables can be chosen in some special way.

The self-similarity of the solutions of partial differential equations either in linear or nonlinear form has allowed their reduction to ordinary differential equations, which often simplifies the investigation. Therefore, with the help of self-similar solutions, researchers and scientists have attempted to envisage the characteristic properties of new phenomena.

Nonlinearity plays a major role in the understanding of most physical, chemical, biological, and engineering sciences. Nonlinear problems fascinate scientists and engineers, but often elude exact treatment. However elusive they may be, the solutions do exist—if only one perseveres in seeking them out.

Although the book does not provide any exercises at the end of each chapter, throughout the book, numerous examples are provided for the appropriate chapter and sections. Thus, the reader will have ample practical examples of dimensional problems instead of facing a cut and dry abstract approach as existing books of this subject follow.

Note that I have tried to make this book stand alone, yet give enough background if the reader has no background on dimensional analysis or the pi theorem at all. Consequently, I have kept Chaps. 1 and 4 of this book similar to what I have published before, which is a book with Springer Publishing Company, under the title *Dimensional Analysis and Self-Similarity Methods for Engineers and Scientists* in 2015.

San Mateo, CA Bahman Zohuri

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I am indebted to the many people who aided me, encouraged me, and supported me beyond my expectations. Some are not around to see the results of their encouragement in the production of this book, yet I hope they know of my deepest appreciations. I especially want to thank my friend Bill Kemp, to whom I am deeply indebted. He has continuously given his support without hesitation and has always kept me going in the right direction.

Above all, I offer very special thanks to my late mother and father and to my children, in particular, my son Sasha. They have provided constant interest and encouragement, without which this book would not have been written. Their patience with my many absences from home and long hours in front of the computer to prepare the manuscript is especially appreciated.

About This Document

The purpose of this document is to describe dimensional analysis, similarity, and modeling methods.

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