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Iconicity and Abduction

Springer

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Introduction

The present book aims to examine the role of iconic signs in scientific hypothesis formation and to show how the relationship between the semiotics of iconicity and the logic of abductive inference finds a natural mathematical setting in category theory and, more specifically, in the categorical theory of topoi and the nonclassical logic of sheaves.

The book's thesis is that the core method of category theory, which lifts properties characterizing individual objects to structural properties characterizing systems of relations linking individuals to one another, helps to illuminate the creative, context-dependent and tentative nature of abductive inference. In particular, category theory sheds new light on how and why mathematics itself is so often successfully employed in scientific hypotheses and their experimental testing.

A helpful touchstone for indicating the parameters and stakes of the present book is Peter Galison's *Image and Logic*, a text that has become something of a minor classic in contemporary philosophy of science. Galison describes his book as being about "the machines of physics" (xvii). Namely, it concerns the complex interplay of social, theoretical and engineering issues in the twentieth-century development of experimental and measurement apparatuses for microphysics, from cloud chambers to supercolliders. Structuring this multilayered genealogy of modern atomic and subatomic physics is the duality of "image" and "logic" indicated by the book's title, the divergence of the evidential criteria in play for the field of physics as a whole between on the one hand photographic images carrying a maximum of relational and relatively unformatted data requiring selective interpretation by observers and on the other hand digitized data such as that provided by particle counters immediately subject to purely logical and mathematical analysis.

What is ultimately at stake for Galison in studying the productive tension between image-based and logic-based data is the theorization of the concrete processes whereby scientific knowledge and the objects of scientific study are co-constructed via hypothesis and experimental testing in local environments (laboratories, seminars, engineering workstations) that must function more or less successfully in their own right as well as globally coordinate their successes appropriately with one another. Galison's is an inquiry into the veritable conditions

of possibility for collective scientific endeavor at the kinds of institutional and financial scales required by contemporary physics, where scientific epistemology is necessarily entwined with political, social, economic and technological concerns. This far-ranging research project has been continued and extended in other works by Galison, notably the collaboration [1] with Lorraine Daston.

In an analogy with Galison's "machines of physics", the present book may be said to concern itself with the "machines of abduction" in scientific inquiry and theorization, particularly its widely prevalent and increasingly sophisticated *mathematical* "machines". Whereas Galison focuses on the *detector* as a type of material apparatus that in twentieth century physics played the role of epistemological bridge "between the microworld and the world of knowledge" (xviii), we examine the Peircean notion of *iconicity* as a semiotic structure linking conceptual fields organized by logical relations on the one hand and real domains organized by structural and causal ones on the other such that the type of linkage itself thereby provides important clues as to the possibility and internal functioning of abductive inference within processes of scientific theory construction. In particular, we aim to show that the semiotics of iconicity as realized in the logic of presheaves and sheaves and rigorously formulated in the language of categorical topoi goes quite some distance in explaining the necessity and the success of the employment of mathematical tools in the scientific investigation of nature.

Like Galison, our primary concern is with the processes whereby scientific knowledge is actually generated. Yet while Galison focuses on the particular field of subatomic physics, we wish to cast a somewhat wider net that hopes to capture essential features of scientific reasoning about the natural world as such. We hope that the apparently unreasonable ambition signaled by this much larger scope of inquiry may perhaps be justified by the fact that we look almost exclusively to the unique role played by mathematics in abductive reasoning in science. We are interested in how the most concrete and situated aspects of scientific hypothesis formation might be modeled in and better understood through techniques and constructions immanent to the abstract world of contemporary mathematics. Iconicity functions as both method and object of investigation here. We subscribe equally to each of the triad of slogans "Mathematical abstraction entails maximum cognitive generality"; "Scientific reasoning concerns what is real and is always itself part of a concrete natural process"; and "Rigorous science requires mathematics".

The overall argument consists of the coherence of three claims: (1) that there is an essential relationship between abductive reasoning and iconicity; (2) that this relationship helps to explain how abductive inferences work; and (3) that category theory and, more particularly, topos theory offers a rich and natural formal domain (a type of mathematical as well as logical "language") for exploring this connection between iconicity and abduction in a rigorous yet also intrinsically abductive way. We take each of these three claims in turn. We claim first of all that there is an important, even essential, relationship between abductive reasoning and iconicity (the semiotic mode whereby one thing signifies or represents something else via a common relational structure); second, that this relationship sheds light on how

abduction works, and in particular helps to explain why abductive inference has remained so resistant to formalization; and finally that category theory, and especially the theory of topoi within it, provides a formal representational environment (a "language") with particular utility for exploring this connection between iconicity and abduction in a mathematically tractable way.

We proceed by examining two philosophers who engage the theoretical problem of the truth/knowledge relation from an ontological standpoint and who make extensive use of mathematical models and techniques: Charles Peirce and Alain Badiou. In their very different work, we identify a common pattern: the difference between Boolean and non-Boolean Heyting algebras and the use of the "geometrical" internalization of logic as a way to expand or modalize a formal situation from within.

Both philosophers use formal systems in an onto-epistemic register. In other words, both of them employ formal languages with the explicit intention of engaging real world processes of reasoning and also of tracking the problem of non-deductive inferences and creative hypotheses. In Peirce, this appears as the coordination of his general theory of semiotics and particularly his notion of diagrammatic mathematics with the iconic logical system of Existential Graphs. In Badiou, this appears as the use of Zermelo–Fraenkel set theory and Paul Cohen's forcing technique to model ontology and abductive inference in *Being and Event*, and his turn in *Logics of Worlds* to the use of sheaves and—in a restricted register—topoi to model a logic of worldly appearance, or "phenomenology". We aim to coordinate these otherwise disparate philosophical approaches within the background theory of mathematical categories.

This book consolidates the co-authors' collaborative work over the past half-dozen or so years. In that time, we have been extremely fortunate to encounter researchers working in related areas whose input, criticisms and suggestions have been invaluable. We should thank first of all our brilliant collaborator at the Center for Diagrammatic and Computational Philosophy, Fernando Tohmé. We thank also Joshua Ramey and Indradeep Ghosh for inviting us to give a workshop at Haverford College in 2012. Gratitude is due as well to the circle of interdisciplinary scholars brought together at the series of Model-Based Reasoning conferences, including especially Woosuk Park, Lorenzo Magnani, and our co-panelists at the Model-Based Reasoning conference at the University of Campinas, Brazil in 2009: Ahti Pietarinen, Jaakko Hintikka and Priscila Borges. We also thank our colleagues at Endicott College, particularly Ellis Cooper, and the college itself for multiple forms of research support in the completing of this project, including one-semester sabbaticals for each of us. Finally, we wish to recognize the helpful comments and suggestions of two anonymous reviewers and the work of the editorial team at Springer/SAPERE.

A synopsis of the content and organization of each of the six chapters may aid the reader in keeping the course of the overall argument in mind while passing first-hand through its details and also to facilitate, if desired, a more pick-and-choose method with respect to specific issues and topics.

Chapter 1 examines the concept of abduction. It traces some of the contemporary lines of investigation into this notion and frames the issue for the present context. We align our approach broadly with the project of naturalized epistemology, that is, treating epistemic agents as real actors in the world subject to all manner of natural conditions. In other words, our view of knowledge is a pragmatic, experimental and fallibilist one. Within this context, however, we are especially interested in scientific knowledge and its prevalent use of mathematics. What kind of experimental activity is the scientific employment of mathematics in the investigation of nature? Because of our focus on this question, the problem of the formalization of abduction becomes particularly acute. We argue that the concept and method of iconicity—a concept that we will try to articulate mathematically through category theory and finally within topoi—provides a way to move forward constructively in addressing the formalization of abductive cognition. We examine several recent formal models to gain some bearings and sketch out a first informal model of the image of iconic epistemology as a functor-like duality, a contravariance, between structured spaces of knowledge and reality. This is the informal intuition that will motivate the mathematical modeling of diagrams as presheaves in the second chapter. Thus, the constructions that follow through the ensuing chapters are motivated essentially by this problem of formalizing abduction as iconic, and its deep intrication of logic, mathematics and epistemology.

Chapter 2 examines Peirce's concept of iconic signs, situating this key concept in the broader framework of Peirce's general theory of semiotics. The chapter culminates in a theory of diagrams as presheaves which serves as the theoretical basis for the ensuing analysis of Peirce's Existential Graph (*EG*) notation.

Chapter 3 shows how Peirce's diagrammatic logical notation of *EG* makes use of iconicity, and how some important features of this iconicity may be represented within category theory. In particular, it is shown how the *alpha* level of the system of *EG* may be represented as a category and how logical conjunction and disjunction correspond to products and coproducts respectively in this category, while material implication may be modeled by categorical exponentiation. This latter fact allows the basic "shape" of abductive inference in classical propositional logic to be characterized by a pair of adjoint functors.

Chapter 4 turns to the work of Alain Badiou and traces his model of truth-procedures as set-theoretical forcing in the manner of Paul Cohen. We interpret this model, and especially its use of generic sets in terms of the problematic of abduction and iconicity in a way that allows for the coordination of Badiou's setting of philosophical ontology in the framework of set theory with Peirce's distinct levels of Existential Graphs.

Chapter 5 examines Badiou's more recent work, *Logics of Worlds*. Here, we specify the notion of sheaves over a Heyting algebra that Badiou uses to model the "phenomenology" of logical relations in and through which the various "objects" of a world are structured together as a unified system. We clarify how Badiou's results may be formulated within topos theory and how this helps to understand the relationship between *Logics of Worlds* and the earlier use of the set-theoretical method of forcing in *Being and Event*.

Finally, Chap. 6 gathers the threads from the previous chapters by showing how topoi provide a natural mathematical setting for the nonstandard "logic" of quantum mechanics. Drawing on the work of Cecilia Flori, Michael Epperson and Elias Zafiris and David Ellerman we show how the formal expression of quantum mechanics in category theory and more specifically sheaf theory and topoi provides a basis for revising—abductively—some of the most fundamental assumptions of scientific realism. This important and far-ranging example drawn from contemporary physics illustrates how the same epistemically abductive structures represented mathematically by presheaves, sheaves and topoi are instantiated also in physical (quantum) systems, thereby opening the path towards a more naturalist interpretation of the abductive models developed in Peirce and Badiou.

A pair of appendices offer brief reminders (or, if necessary, lean introductions) for the reader of the essential elements of category theory and topos theory. Parts of chapters two, three and four have appeared previously as articles in *Synthese, Logic Journal of IGPL* and *Studies in Computational Intelligence*. For all of their gracious support, this book is dedicated to our wives and sons.

Reference

L. Daston, P. Galison, *Objectivity* (Zone Books, Brooklyn, 2007)