

# **Advanced Textbooks in Control and Signal Processing**

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Alberto Isidori

# Lectures in Feedback Design for Multivariable Systems

 Springer

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## Series Editors' Foreword

The *Advanced Textbooks in Control and Signal Processing* series is designed as a vehicle for the systematic textbook presentation of both fundamental and innovative topics in the control and signal processing disciplines. It is hoped that prospective authors will welcome the opportunity to publish a more rounded and structured presentation of some of the newer emerging control and signal processing technologies in this textbook series. However, there will always be a place in the series for contemporary presentations of foundational material in these important engineering areas.

The science of control comprises two interdependent activities: control theory and control engineering. Control theory provides an understanding of how and why controllers work, while control engineering is concerned with transforming theoretical solutions into controller designs that can be implemented in real applications. Sometimes it is control theory that leads the way, suggesting new methods or providing a deeper understanding of how to solve control problems. At other times it is an engineering solution or a technical innovation that creates the means of doing control differently, or opens up a new applications field, and then control theorist has to catch up. Either way control has been a very vibrant and expanding field for engineers and researchers since the middle of the last century. It has been a fascinating journey that continues today, exploiting links to areas in computer science and networks.

Alberto Isidori has traveled this path not only as an eminent control theorist and engineer, but also as a professional, contributing to the development of the important international institutions of the control community. In many ways this set of lectures is his personal reflection on the methods of feedback design for multivariable systems. He seeks to present an overarching view of design for both linear and nonlinear multivariable systems, identifying structural similarities within the two forms. A linear form can of course be viewed as a special case of a nonlinear system and it should not be surprising that strong theoretical links exist between the two.

Also found in this textbook is some work on coordinated control in what are often termed “leader–follower” systems. It is the introduction of restricted

information transfer between the participating systems that makes the control design challenging. Leader–follower systems and the coordinated control design problem are one continuing interest at today's international control conferences.

The insightful, reflective nature of the textbook assumes that the reader has already studied the basics of control theory. A guide to the knowledge that is assumed of the reader can be found in the two useful appendices.

The text is based on Prof. Isidori's considerable teaching experience, including many years of lecturing at the Sapienza University in Rome, and at various international guest lecture courses. It provides a thoughtful and insightful exposition of where we stand today in control theory and is a very welcome and valuable addition to the *Advanced Textbooks in Control and Signal Processing* series.

## About the Author

**Alberto Isidori** has been a Professor of Automatic Control at the University Sapienza, Rome since 1975. He has been a visiting academic at most of the foremost international universities and institutions where control theory and control engineering is preeminent as a subject of study.

He was President of the European Union Control Association (EUCA) between 1995 and 1997. And for the period 2008–2011, he was President of the International Federation of Automatic Control (IFAC).

Professor Isidori is noted for his books on nonlinear systems some of which were published by Springer, notably:

- *Nonlinear Control Systems* by Alberto Isidori (ISBN 978-3-540-19916-8, 1995)
- *Nonlinear Control Systems II* by Alberto Isidori (ISBN 978-1-85233-188-7, 1999); and
- *Robust Autonomous Guidance* by Alberto Isidori, Lorenzo Marconi and Andrea Serrani (ISBN 978-1-85233-695-0, 2003); a volume in the *Advances in Industrial Control* monograph series.

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# Preface

The purpose of this book is to present, in organized form, a number of “selected topics” in feedback design, for linear and nonlinear systems. The choice of topics reflects a teaching activity held in the past 15 years at the University Sapienza in Rome as well as at various other academic institutions. The focus of the book is on methods for achieving asymptotic stability, and disturbance rejection, in the presence of model uncertainties. Among various possible options to deal with such design problem, the emphasis is on methods that, in one form or another, appeal to the classical “small-gain” principle for robust stability. In this setting, the aim is to offer a presentation as much as possible similar for linear and nonlinear systems. Of course, for pedagogical reason, linear systems are handled first. Even though the focus of the book is multi-input multi-output systems, for pedagogical reasons, single-input single-output nonlinear systems are handled first in some detail. As a result, the book begins with a rather tutorial flavor (Chaps. 2 to 8) and ends with a more monographic nature (Chaps. 9 to 12). A more detailed description of the topics covered in the book can be found in the introductory Chap. 1. The book presumes some familiarity with the theory of linear systems and with a few basic concepts associated with the analysis of the stability of equilibrium in a nonlinear system. For the reader’s convenience, a sketchy summary of the relevant background concepts is offered in the two appendices.

The monographic portion of the book reflects in part my own research activity, conducted in collaboration with Lorenzo Marconi and Laurenty Praly, whose unselfish friendship, competence, and patience is gratefully acknowledged. The topics covered in this book have been taught during the past few years at the University Sapienza in Rome as well as at the EECI Graduate School on Control, in Paris, and at the Institute of Cybersystems and Control of Zhejiang University, in

Hangzhou. Encouragement and support from Françoise Lamnabhi and Hongye Su, respectively, are gratefully acknowledged. In particular, while at Zhejiang University, I have been able to establish a fruitful collaboration with Lei Wang, the outcome of which is reflected in the material presented in Chaps. 10 and 11.

Rome, Italy  
May 2016

Alberto Isidori



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