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Statistical Theory of Heat

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Preface

The theory of heat plays a peculiar and outstanding role in theoretical physics. Because of its general validity, it serves as a bridge between rather diverse fields such as the theory of condensed matter, elementary particle physics, astrophysics and cosmology. In its classical domain, it describes primarily averaged properties of matter, starting with systems containing a few particles, through aggregate states of ordinary matter around us, up to stellar objects, without direct recourse to the physics of their elementary constituents or building blocks. This facet of the theory carries far into the description of condensed matter in terms of classical physics. In its statistical interpretation, it encompasses the same topics and fields but reaches deeper and unifies classical statistical mechanics with quantum theory of many-body systems.

In the first chapter, I start with some basic notions of thermodynamics and introduce the empirical variables which are needed in the description of thermodynamic systems in equilibrium. Systems of this kind live on low-dimensional manifolds. The thermodynamic variables, which can be chosen in a variety of ways, are coordinates on these manifolds. Definitions of the important thermodynamical ensembles, which are guided by the boundary conditions, are illustrated by some simple examples.

The second chapter introduces various thermodynamic potentials and describes their interrelation via Legendre transformations. It deals with continuous changes of states and cyclic processes which illustrate the second and third laws of thermodynamics. It concludes with a discussion of entropy as a function of thermodynamic variables.

The third chapter is devoted to geometric aspects of thermodynamics of systems in equilibrium. In a geometric interpretation, the first and second laws of thermodynamics take a simple and transparent form. In particular, the notion of latent heat, when formulated in this framework, becomes easily understandable.

Chapter 4 collects the essential notions of the statistical theory of heat, among them *probability measures* and *states* in statistical mechanics. The latter are illustrated by the three kinds of statistics, the classical, the fermionic and the bosonic statistics. Here, the comparison between classical and quantum statistics is particularly instructive.

Chapter 5 starts off with phase mixtures and phase transitions, treated both in the framework of Gibbs' thermodynamics and with methods of statistical mechanics.

Finally, a last, long section of this chapter, as a novel feature in a textbook, discusses the problem of stability of matter. We give a heuristic discussion of an intricate analysis that was developed fairly late, about half a century after the discovery of quantum mechanics.

I am very grateful to the students whom I had the privilege to guide through their “years of apprenticeship”, to my collaborators and to many colleagues for questions, comments and new ideas. Among the latter, I thank Rolf Schilling for advice. Also, I owe sincere thanks to Andres Reyes Lega who read the whole manuscript and made numerous suggestions for improvement and enrichment of the book.

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Mainz, Germany
August, 2016

Florian Scheck

Contents

1	Basic Notions of the Theory of Heat	1
1.1	Introduction	1
1.2	First Definitions and Propositions	1
1.3	Microcanonical Ensemble and Ideal Gas	9
1.4	The Entropy, a First Approach	12
1.5	Temperature, Pressure and Chemical Potential	18
1.5.1	Thermal Contact	18
1.5.2	Thermal Contact and Exchange of Volume	23
1.5.3	Exchange of Energy and Particles	24
1.6	Gibbs Fundamental Form	25
1.7	Canonical Ensemble, Free Energy	27
1.8	Excursion: Legendre Transformation of Convex Functions	30
2	Thermodynamics: Classical Framework	39
2.1	Introduction	39
2.2	Thermodynamic Potentials	39
2.2.1	Transition to the Free Energy	39
2.2.2	Enthalpy and Free Enthalpy	40
2.2.3	Grand Canonical Potential	42
2.3	Properties of Matter	46
2.4	A Few Thermodynamic Relations	49
2.5	Continuous Changes of State: First Examples	50
2.6	Continuous Changes of State: Circular Processes	59
2.6.1	Exchange of Thermal Energy Without Work	59
2.6.2	A Reversible Process	61
2.6.3	Periodically Working Engines	62
2.6.4	The Absolute Temperature	65
2.7	The Laws of Thermodynamics	66
2.8	More Properties of the Entropy	72
3	Geometric Aspects of Thermodynamics	75
3.1	Introduction	75
3.2	Motivation and Some Questions	75

3.3	Manifolds and Observables	77
3.3.1	Differentiable Manifolds	77
3.3.2	Functions, Vector Fields, Exterior Forms	79
3.3.3	Exterior Product and Exterior Derivative	82
3.3.4	Null Curves and Standard Forms on $\mathbb{R}^n$	87
3.4	The One-Forms of Thermodynamics	90
3.4.1	One-Forms of Heat and of Work	91
3.4.2	More on Temperature	92
3.5	Systems Depending on Two Variables Only	95
3.6	*An Analogy from Mechanics	100
4	Probabilities, States, Statistics	105
4.1	Introduction	105
4.2	The Notion of State in Statistical Mechanics	105
4.3	Observables and Their Expectation Values	111
4.4	Partition Function and Entropy	114
4.5	Classical Gases and Quantum Gases	123
4.6	Statistics, Quantum and Non-quantum	129
4.6.1	The Case of Classical Mechanics	129
4.6.2	Quantum Statistics	130
4.6.3	Planck's Radiation Law	134
5	Mixed Phases, Phase Transitions, Stability of Matter	141
5.1	Introduction	141
5.2	Phase Transitions	141
5.2.1	Convex Functions and Legendre Transformation	142
5.2.2	Phase Mixtures and Phase Transitions	151
5.2.3	Systems with Two or More Substances	156
5.3	Thermodynamic Potentials as Convex or Concave Functions	159
5.4	The Gibbs Phase Rule	161
5.5	Discrete Models and Phase Transitions	163
5.5.1	A Lattice Gas	163
5.5.2	Models of Magnetism	165
5.5.3	One-Dimensional Models with and Without Magnetic Field	169
5.5.4	Ising Model in Dimension Two	172
5.6	Stability of Matter	178
5.6.1	Assumptions and First Thoughts	179
5.6.2	Kinetic and Potential Energies	182
5.6.3	Relativistic Corrections	185
5.6.4	Matter at Positive Temperatures	191

6 Exercises, Hints and Selected Solutions	197
Literature	229
Index.....	231