Statistical Analysis of Noise in MRI

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Modeling, Filtering and Estimation

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"How do you peel a porcupine?"

Foreword

Medical imaging and the field of radiology have come a long way since Wilhelm Röntgen's discovery of the X-ray in 1895. Medical imaging is today an integral part of modern medicine and includes a large number of modalities such as X-ray computed tomography (CT), ultrasound, positron emission tomography (PET), and magnetic resonance imaging (MRI).

This book, "Statistical Analysis of Noise in MRI," presents a modern signal processing approach for medical imaging with a focus on noise modeling and estimation for MRI. MRI scanners use strong magnetic fields, radio waves, and magnetic field gradients to form images of the body. MRI has seen a tremendous development during the past four decades and is now an indispensable part of diagnostic medicine. MRI is unparalleled in the investigation of soft tissues due to its superior contrast sensitivity and tissue discrimination.

I met the lead author of this book Dr. Santiago Aja-Fernández for the first time in 2006 when he was a visiting Fulbright scholar in my laboratory, Laboratory of Mathematics in Imaging at Brigham and Women's Hospital, Harvard Medical School, Boston. His goal was clear from the beginning: to learn more about MRI. His plan was to combine this knowledge with his then already vast knowledge about statistical signal processing. He had a very productive year in Boston and subsequently published several, now well-cited, papers on noise estimation in MRI.

During Santiago's year-long visit in my laboratory we were investigating the boundaries of what it meant to separate signals from noise. What do you need to know about the data to do this well? The more complicated the image formation process is, the less the commonly assumed model that the noise is Gaussian is applicable. This book is about exploring these questions and providing guidelines on how to proceed. One important message in this book is that you have to understand your data acquisition in detail. Santiago Aja-Fernández continued to work on these questions when he returned to the University of Valladolid with the second author of this book, Dr. Gonzalo Vegas-Sánchez-Ferrero. They and their co-workers have made tremendous progress during the past decade and have become authorities on the topic of noise modeling in MRI.

I expect that the importance of accurate noise modeling and estimation in the field of MRI will increase over the next several years due to the increasing complexity of the MRI scanners. Many commercial scanners now have the possibility to connect multiple RF detector coil sets to allow the simultaneous acquisition of several signals in a phased array system. These systems were originally developed to reduce the scanning time and therefore to avoid some problems with moving structures, as well as to enhance the signal-to-noise ratio of the magnitude image. Noise modeling is important in noise removal, but perhaps even more so when estimating derived parameters from this more complex measured data. For example, robust estimation of the diffusion tensor in diffusion MRI requires in-depth knowledge of the imaging process used for creating the multi-channel diffusion MRI data. With today's complex parallel imaging acquisition schemes commonly used in the clinic, it is important to be able to understand how to model the data appropriately for any subsequent signal processing task.

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Preface

This work is the result of more than 10 years of research in the area of MRI from a signal and noise perspective. Our interest has always been to properly model the noise that affects our signals, in order to design the best possible algorithms based on that knowledge. All this time we have found many great works that were coming along with our own research, offering alternative points of view. We realized that most of the works dealing with noise in MRI can be seen as complementary efforts rather than competitive. It was necessary, thus, to systematize all that knowledge that had arisen, in order to understand the problem as a whole. It is precisely in the relations between distinct methods and philosophies where the real nature of this question can be better understood. In this work we gather different approaches to noise analysis in MRI, systematizing and classifying the different methods, trying to bring them together to common ground. So, instead of being seen as independent efforts, they can be considered as consecutive paces along the same way.

This book is intended to serve as a reference manual for researchers dealing with signal processing in MRI acquisitions. It is written from a signal theory perspective, using probabilistic modeling as a basic tool. Readers are assumed to know the basic principles of linear systems and signal processing, as well as being familiar with random variables, image processing, and calculus fundaments. It could also serve as a textbook for postgraduate students in engineering with an interest in medical image processing.

We provide a complete framework to model and analyze noise in MRI, considering different modalities and acquisition techniques, focusing on three issues: noise modeling, noise estimation, and noise filtering. To that end, the book is divided into three parts. The first part analyzes the problem of noise in MRI, the modeling of the acquisition, and the definition of the most common statistical distributions used to describe the noise. The problem of noise and signal estimation for medical imaging is analyzed from a statistical signal processing perspective. The second part of the book is devoted to analyzing and reviewing the different techniques to estimate noise out of a single MRI slice in single- and multiple-coil systems for fully sampled acquisitions. The third part deals with the problem of noise estimation when accelerated acquisitions are considered and parallel imaging methods are used to reconstruct the signal. The book is complemented with three appendices.

Our intention is to make the book comprehensive, thus many definitions and methods have been included, and some ideas are repeated in different chapters from different perspectives. That way, most of the chapters can be understood independently of the others, although relations between them will always be present. Some theoretical topics about random variables, image processing, and MRI acquisition have been omitted for the sake of compactness. We provide a complete bibliography that can be used to fill the gaps.

Finally, note that this is a field of constant expansion, with new methods being published every year. In addition, acquisition techniques are also rapidly evolving, producing new models of noise that are not analyzed here. We consider this book as the framework that could serve as the basis for the analysis of all those novelties that will surely arise in the next years.

Valladolid, Spain Santiago Aja-Fernández
March 2016

Acknowledgments

The work presented in this book started at LMI (Harvard Medical School, Boston) almost 10 years ago, funded by a Fulbright Scholarship. Many different researchers have contributed to the development of the main corpus on noise modeling and estimation that is finally gathered here. In particular, I want to thank Dr. Tristán-Vega for all the shared work in this field and to my coauthor, Gonzalo Vegas-Sánchez-Ferrero, for his help and support in the elaboration of this book. Let us hope we can work in new topics in the future. The other researchers that have actively contributed with their knowledge are Prof. C.F. Westin, Prof. Alberola-López, Dr. K. Krissian, Dr. M. Niethammer, Dr. V. Brion, and Dr. W.S. Hoge.

Our intent to make a comprehensive book implies a great amount of work that could not have been done without external support from other researchers. I specially want to thank Tomasz Pieziak, from AGH University of Science and Technology, Krakow (Poland), whose work about VST is directly used in this book. We use some parts of his Ph.D. thesis for the chapter about blind estimation, and he was also a great help in the implementation of some of the methods for comparison. The filtering chapter takes many references from Dr. Veronique Brion's Ph.D. thesis, to whom I must be very grateful for saving me a great amount of time.

The data used in this book come from different sources, but I want to thank Dr. W. Scott Hoge and Dr. Diego Hernando for providing the valuable raw data used along the book for validation. Additional scanning was done in Q-Diagnóstico (Valladolid) and the 3T- scanner of Instituto de Técnicas Intrumentales (Universidad de Valladolid). We also use an ilustration taken from Dr. Tristán-Vega's thesis that was generated using HARDI data kindly provided by the Australian eHealth Research Centre-CSIRO ICT Centre, Brisbane (Australia).

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Last but not least, I am in great debt with my wife Isabel and my child Juan, from whom I steal the many hours I dedicated to the writing of this book. I will not forget you when I become rich and famous.

March 2016 Santiago Aja-Fernández

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Acronyms and Notation

Acronyms

ACS	Auto Calibration Signal
ADC	Apparent Diffusion Coefficient
ARC	Autocalibrating Reconstruction of Cartesian imaging
ASL	Arterial Spin Labeling
ASSET	Array coil Spatial Sensitivity Encoding
AWGN	Additive White Gaussian Noise
BOLD	Blood Oxygen Level Dependent
c-χ	Central chi
CA	Conventional Approach
CAIPIRINHA	Controlled Aliasing in Parallel Imaging Results in Higher Acceleration
CHARMED	Composite Hindered and Restricted Model of Diffusion
CMS	Composite Magnitude Signal
CURE	Chi-square Unbiased Risk Estimator
CV	Coefficient of Variation
DCT	Discrete Cosine Transform
DFT	Discrete Fourier Transform
DKI	Diffusion Kurtosis Imaging
DoF	Degrees of Freedom
DOT	Diffusion Orientation Transform
DT	Diffusion Tensor
DTI	Diffusion Tensor Imaging
DWI	Diffusion Weighted Imaging
DWT	Discrete Wavelet Transform
EM	Expectation Maximization
EPI	Echo Planar Imaging
FFE	Fast Field Echo

fMRI	Functional Magnetic Resonance Imaging
FOV	Field of View
GRAPPA	Generalized Autocalibrating Partially Parallel Acquisition
HARDI	High Angular Resolution Diffusion Image
HMF	Homomorphic Filter
iDFT	Inverse Discrete Fourier Transform
IID	Independent and Identically Distributed
KDE	Kernel Density Estimator
LLS	Linear Least Squares
LMMSE	Linear Minimum Mean Square Error
LPF	Low-Pass Filter
LS	Least Squares
MAD	Median Absolute Deviation
MAP	Maximum a Posteriori
MGF	Moment generating function
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MR	Magnetic Resonance
MRI	Magnetic Resonance Imaging
MRV	Markov Random Field
nc-χ	Non-central chi
NEX	Number of Excitations
NLM	Non-local Means
NLS	Nonlinear Least Squares
NMR	Nuclear Magnetic Resonance
ODF	Orientation Density Function
OPDF	Orientation Probability Density Function
OSRAD	Oriented Rician Noise-Reducing Anisotropic Diffusion
PCA	Principal Component Analysis
PD	Proton Density
PDE	Partial Differential Equation
PDF	Probability Density Function
pMRI	Parallel MRI
RE	Relative Error
RF	Radio Frequency
RMMSE	Recursive Linear Minimum Mean Square Error
ROI	Region of interest
RV	Random Variable
SENSE	Sensitivity Encoding for Fast MRI
SLV	Sample Local Variance
SMASH	Simultaneous Acquisition of Spatial Harmonics
SMF	Spatial Match Filter
SNR	Signal-to-Noise Ratio
SoS	Sum of Squares
SRRAD	Scalar Rician Noise-Reducing Anisotropic Diffusion

STD	Standard Deviation
SVD	Singular Value Decomposition
SWT	Stationary Wavelet Transform
TR	Repetition Time
TSE	Turbo Spin Echo
TV	Total Variation
UNLM	Unbiased Non-local Means
VST	Variance Stabilization Transform
WLS	Weighted Least Squares

Notation

Probability, Estimation and Moments

$p_X(x)$	Probability density function of X
$E\{X\}$	Expectation of random variable X
$E\{X^p\}$	Order p moment of random variable X
$\text{Var}\{X\}$	Variance of random variable X
$\text{std}\{X\}$	Standard deviation of random variable X
$\text{CV}\{X\}$	Coefficient of variation of random variable X $\text{CV}\{X\} = \frac{\text{std}\{X\}}{E\{X\}}$
$\langle M(\mathbf{x})\rangle$	(Global) Sample mean of image $M(\mathbf{x})$ $\langle M(\mathbf{x})\rangle = \frac{1}{\lvert\Omega\rvert}\sum_{\mathbf{x}\in\Omega} M(\mathbf{x})$
$\langle M(\mathbf{x})\rangle_{\mathbf{x}}$	Local sample mean of image $M(\mathbf{x})$ $\langle M(\mathbf{x})\rangle_{\mathbf{x}} = \frac{1}{\lvert\eta(\mathbf{x})\rvert}\sum_{\mathbf{p}\in\eta(\mathbf{x})} M(\mathbf{p})$ with $\eta(\mathbf{x})$ a neighborhood centered in $\mathbf{x}$
$\langle M(\mathbf{x})\rangle_{\mathbf{k}}$	Local sample mean of image $M(\mathbf{x})$ calculated along N samples $\langle M(\mathbf{x})\rangle_{\mathbf{k}} = \frac{1}{N}\sum_{k=1}^{N} M_k(\mathbf{x})$
$\mathcal{V}(M(\mathbf{x}))$	(Global) sample variance of $M(\mathbf{x})$ $\mathcal{V}(M(\mathbf{x})) = \langle M^2(\mathbf{x})\rangle - \langle M(\mathbf{x})\rangle^2$
$\mathcal{V}_{\text{x}}(M(\mathbf{x}))$	Sample local variance of $M(\mathbf{x})$ $\mathcal{V}(M(\mathbf{x}))_{\mathbf{x}} = \langle M^2(\mathbf{x})\rangle_{\mathbf{x}} - \langle M(\mathbf{x})\rangle_{\mathbf{x}}^2$
$\text{median}(X)$	Median of random variable X
$\text{mode}\{I(\mathbf{x})\}$	Mode of the distribution of $I(\mathbf{x})$
$\widehat{a}$	Estimator of parameter a

Regions and Topology

$M(\mathbf{x}_\mathrm{B})$	Background area of image $M(\mathbf{x})$ $\mathbf{x}_\mathrm{B} = \mathbf{x} \vert A(\mathbf{x}) = 0$
$M(\mathbf{x}_\mathrm{R})$	$M(\mathbf{x})$ in the region $\boldsymbol{R}$ $\mathbf{x}_\mathrm{R} \in \boldsymbol{R}$

Operators

$\mathcal{F}\{S(\mathbf{x})\}$	Fourier transform of $S(\mathbf{x})$
$\mathcal{F}^{-1}\{s(\mathbf{k})\}$	Fourier inverse transform of $s(\mathbf{k})$
$\mathrm{LPF}(S(\mathbf{x}))$	Low-pass filter of signal $S(\mathbf{x})$
MAD	Median absolute deviation $\mathrm{MAD}(g_i) = \underset{i}{\mathrm{median}}\left(\left\Vert g_i - \underset{k}{\mathrm{median}}(g_k) \right\Vert \right)$
$\mathrm{MAD}_\mathbf{x}$	Local median absolute deviation $\mathrm{MAD}_\mathbf{x}(S(\mathbf{x})) = \underset{\mathbf{p}\in\eta(\mathbf{x})}{\mathrm{median}} \left\vert S(\mathbf{p}) - \underset{\mathbf{q}\in\eta(\mathbf{x})}{\mathrm{median}}(S(\mathbf{q})) \right\vert$
$\xi(\theta)$	Koay correction factor $\xi(\theta) = 2 + \theta^2 - \frac{\pi}{8} \cdot e^{-\frac{\theta^2}{2}} \cdot \left[(2+\theta^2) I_0\left(\frac{\theta^2}{4}\right) + \theta^2 I_1\left(\frac{\theta^2}{4}\right) \right]^2$
$f_\mathrm{stab}()$	Variance stabilization transform
$\circledast$	(Circular) convolution
div	Divergence operator
∇	Gradient operator

Matrix Operations

$\mathrm{tr}(\mathbf{C})$	Trace of matrix $\mathbf{C}$ $\mathrm{tr}(\mathbf{C}) = \sum_i c_{i,i}$ with $c_{i,i}$ the diagonal elements of matrix $\mathbf{C}$
$\Vert\mathbf{C}\Vert_1$	L_1 norm $\Vert\mathbf{C}\Vert_1 = \sum_i \sum_j \left\vert c_{i,j} \right\vert$ with $c_{i,j}$ the elements of matrix $\mathbf{C}$
$\Vert\mathbf{C}\Vert_F$	Frobenius norm $\Vert\mathbf{C}\Vert_F = \sqrt{\sum_i \sum_j \left\vert c_{i,j} \right\vert^2} = \sqrt{\mathbf{C}\mathbf{C}^H}$ with $c_{i,j}$ the elements of matrix $\mathbf{C}$
$\mathbf{C}^H$	Conjugate transpose of matrix $\mathbf{C}$
$\mathbf{C}^{-1}$	Inverse of matrix $\mathbf{C}$

Functions

$I_n(x)$	Modified Bessel Function of the first kind of order n
$L_n(x)$	Laguerre polymonial of order n $L_n(x) = {}_1F_1(-n; 1; x)$

${}_1F_1(a;b;x)$	Confluent hypergeometric function of the first kind
$\Gamma(n)$	Gamma function
$u(x)$	Heaviside step function
$\mathrm{erf}(x)$	Error function $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$