Undergraduate Texts in Mathematics

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Undergraduate Texts in Mathematics are generally aimed at third- and fourth-year undergraduate mathematics students at North American universities. These texts strive to provide students and teachers with new perspectives and novel approaches. The books include motivation that guides the reader to an appreciation of interrelations among different aspects of the subject. They feature examples that illustrate key concepts as well as exercises that strengthen understanding.

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Kristopher Tapp

Differential Geometry of Curves and Surfaces

Springer

Kristopher Tapp
Department of Mathematics
Saint Joseph's University
Philadelphia, PA, USA

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Introduction

About Differential Geometry

Differential geometry uses calculus to study curved shapes, such as the trajectory of a missile, the shape of an airplane wing, and the curvature of Einstein's spacetime. The story begins with characters familiar from multivariable calculus: parametrized curves and surfaces. Along the way, techniques from calculus, linear algebra, and real analysis are blended together, often within a single proof. All of the material is visually grounded with substantial motivation. In the end, the various prerequisite topics will not seem so different from each other or from geometry. Looking forward, differential geometry prepares the reader for a sizable portion of modern physics and mathematics.

About This Book

The differential geometry of curves and surfaces is a well-developed topic about which volumes have been written. My goal is to engage the modern reader with clear and colorful explanations of the essential concepts, culminating in the famous Gauss–Bonnet Theorem.

I wrote this book to serve a variety of readers. For readers seeking an elementary text, I kept the prerequisites minimal, included plenty of examples and intermediate steps within proofs, and clearly identified as optional the more excursive applications and the more advanced topics. For readers bound for graduate school in math or physics, this is a clear, concise, rigorous development of the topic including the deep global theorems. For the benefit of *all* readers, I employed every trick I know to render the difficult abstract ideas herein more understandable and engaging.

Over 300 color illustrations bring the mathematics to life, instantly clarifying concepts in ways that grayscale could not. Green-boxed definitions and purple-boxed theorems help to visually organize the mathematical content.

Applications abound! The study of conformal and equiareal functions is grounded in its application to cartography. Evolutes, involutes, and cycloids are introduced through Christiaan Huygens's fascinating story. He attempted to solve the famous longitude problem with a mathematically improved pendulum clock. Along the way, he invented mathematics that would later be applied to optics and gears. Clairaut's theorem is presented as a conservation law for angular momentum. Green's theorem makes possible a drafting tool called a planimeter. Foucault's pendulum helps one visualize a parallel vector field along a latitude of the Earth. Even better, a south-pointing chariot helps one visualize a parallel vector field along any curve in any surface.

In truth, the most profound application of differential geometry is to modern physics, which is beyond the scope of this book. The GPS in my car wouldn't work without general relativity, formalized through the language of differential geometry. The above-mentioned applications don't purport to match the significance of modern physics, but instead they serve a crucial pedagogical role within this book: to ground each abstract idea in something concrete. Search YouTube for "south-pointing chariot" and you will learn about a fascinating toy, but in this book it's more than that—it's a concrete device that buttresses the rigorous definition of a parallel vector field and motivates the variational formulas for arc length. Throughout this book, applications, metaphors, and visualizations are tools that motivate and clarify the rigorous mathematical content, but never replace it.

To emphasize geometric concepts, I have systematically put local coordinate formulas in their proper place: near the end of each chapter. These formulas empower the reader to compute various curvature measurements in particular examples, but they do not define these measurements. Local coordinate formulas are necessary for certain proofs, but it's amazing how much can be done without them. To me, differential geometry represents a beautiful interwoven collection of visual insights, made rigorous with a bit of calculus and linear algebra. My goal is make it as easy as possible for readers to internalize the proofs and the intuitions that support them.

Prerequisites

This text requires only a minimal one-semester course in each of the following three prerequisite topics:

(1) Multivariable calculus (not necessarily including Green's theorem)
(2) Linear algebra
(3) Real analysis (not necessarily including multivariable content)

Along the way, I have included brief overviews of some of this prerequisite content, plus an appendix covering topology (continuity, connectedness, and compactness) at a pace that assumes some previous exposure.