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Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

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Hak-Keung Lam

Polynomial Fuzzy Model-Based Control Systems

Stability Analysis and Control Synthesis
Using Membership Function-Dependent
Techniques

 Springer

Hak-Keung Lam
Department of Informatics
King's College London
London
UK

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*To my family, my beloved wife,
Esther Wing See Chan, and
my lovely daughter, Katrina Faye Lam,
for standing by me*

Hak-Keung Lam

Preface

I have researched on fuzzy model-based control systems since 1995 as a Ph.D. student. During the past two decades, I have gained a rich experience and knowledge in the field, and have summarized the research achievements by a number of publications.

I have witnessed the development of the research in the field of fuzzy model-based control systems in the past 20 years. In general, I divide it into four stages. In this first stage, from early 1970s to late 1980s, fuzzy control has become well known as an intelligent control strategy for ill-defined and complex systems due to successful applications from household appliances to chemical plants. Using fuzzy logic concept, human spirit can be captured by linguistic rules which can be realized by machines. Fuzzy logic controller is thus able to incorporate human knowledge to control complex systems. In the early applications, the fuzzy logic controller was designed heuristically without the need of the mathematical model of nonlinear systems. Although good performance can be demonstrated by some application examples, essential issues such as system stability and robustness are not guaranteed which put the users and applications at risk.

In the second stage, from early 1990s to mid-2000s, thanks to the T-S fuzzy model fuzzy model-based control has become very popular and offered a systematic way for system analysis and control design. Stability analysis has become a very promising research topic since then. Fruitful analysis results have been reported in many articles. Relaxation of stability conditions has drawn a great deal of attention from the researchers in the fuzzy control community. As the stability analysis has not considered the membership functions in most of the work during this period, the stability analysis/conditions are named as membership function-independent stability analysis/conditions in my publications.

The third stage was from mid-2000s to late 2000s. I have proposed the membership function-dependent stability analysis which is able to bring the information and characteristic of the membership functions into the stability conditions. Consequently, it is named as membership function-dependent stability conditions in my publications. As the membership functions are the nonlinearity of the nonlinear

system, it plays an important role to achieve more relaxed stability analysis results compared with the membership function-dependent stability analysis. Furthermore, opposite to the concept of parallel distributed compensation, I have promoted the concept of partially/imperfectly matched premises that the number of rules and/or premise membership functions used in the fuzzy controller are different from those of the fuzzy model to achieve greater control design flexibility and lower control implementation complexity (to reduce the implementation costs).

The fourth stage started from late 2000s. The introduction of the polynomial fuzzy model takes the stability analysis and fuzzy control to another level using sum-of-squares approach instead of linear matrix inequality.

This book focuses on the work on the fourth stage which is the research on the stability analysis of polynomial fuzzy model-based control systems where the concept of partially/imperfectly matched premises and membership function-dependent analysis are considered. I would like to summarize my recent achievements on this topic which present the latest research outcomes including findings, observations, concepts, ideas, research directions, stability analysis techniques, and control methodologies. The membership function-dependent analysis offers a new research direction for the fuzzy model-based control systems by taking into account the characteristic and information of the membership functions (related to the nonlinearity of the plant) in the stability analysis. Membership function-dependent stability conditions are far more relaxed compared with some state-of-the-art membership function-independent stability conditions. It is more effective to deal with nonlinear control problems as membership function-dependent approach considers the dedicated nonlinear system on hand rather than a family of nonlinear systems tackled in the membership function-independent approach. Through this book, I would like to promote the membership function-dependent analysis to be a new research direction and hope to see that it becomes a popular technique to deal with the stability analysis problem for fuzzy model-based control systems.

The content of this book is mainly at the research level presenting the most recent and advanced research results, which aims to promote the research of polynomial fuzzy model-based control systems, provide theoretical support, and point a research direction to postgraduate students and fellow researchers. The introduction and preliminary parts of the book provides an overview of the topics and technical materials are presented in a very detailed manner. Numerical examples are provided in each chapter to verify the analysis results, demonstrate the effectiveness of the proposed polynomial fuzzy control schemes, and explain the design procedure. This book is comprehensively written with detailed derivation steps and mathematical details to enhance the reading experience, in particular, for readers without extensive knowledge on the topics. It is thus also recommended to undergraduate students with control background who are interested in polynomial fuzzy model-based control systems.

This book has four parts consisting of ten chapters. The first part Introduction and Preliminaries provides the overview and technical background of the fuzzy model-based control systems offering fundamental knowledge and mathematical support for the subsequent parts. The second part Stability Analysis Techniques

presents the latest techniques based on the membership function-dependent stability analysis for polynomial fuzzy model-based control systems. The third part Advanced Control Methodologies extends the stability analysis techniques to more challenging control problems. The fourth part Advanced Lyapunov Functions introduces more effective Lyapunov functions for stability analysis and polynomial fuzzy control strategy for the control of nonlinear plants. The content of each chapter is briefly introduced below.

Part I Introduction and Preliminaries

- Chapter 1 gives a general overview of the fuzzy model-based control which covers the background, literature review, development of the field, fuzzy models, fuzzy control methodologies, stability analysis approaches, and control problems.
- Chapter 2 provides the technical and mathematical background for the fuzzy model-based control which offers the equations of the fuzzy model and closed-loop systems, definition of variables, published stability conditions in terms of linear matrix inequalities, and sum of squares (SOS). These materials are essential for the work in the subsequent chapters.

Part II Stability Analysis Techniques

- Chapter 3 investigates the stability of polynomial fuzzy model-based control systems by treating the membership functions and system states as symbolic variables. The information of membership functions is considered in the stability analysis and brought to the SOS-based stability conditions. Techniques are proposed to introduce slack matrix variables carrying the information of membership functions to the SOS-based stability conditions without increasing much the computational demand.
- Chapter 4 investigates the stability of polynomial fuzzy model-based control systems by bringing the approximated membership functions into the SOS-based stability conditions. Various approximation methods of membership functions are reviewed and their characteristics are discussed. Using the Taylor series expansion, the original membership functions are represented by approximated membership functions which are a weighted sum of local polynomials in a favorable form for stability analysis. SOS-based stability conditions are obtained which guarantee the system stability if the fuzzy model-based control system is stable at all chosen Taylor series expansion points.
- Chapter 5 investigates the stability of general polynomial fuzzy model-based control systems. In Chaps. 3 and 4, a constraint that the polynomial Lyapunov function matrix is allowed to be dependent on some state variables determined by the structure of the input matrices is required to obtain convex stability

conditions. In this chapter, this constraint is removed and a two-step procedure is proposed to search for a feasible solution to the SOS-based stability conditions. Consequently, the stability analysis results can be applied to a wider range of polynomial fuzzy model-based control systems.

Part III Advanced Control Methodologies

- Chapter 6 considers a regulation problem for polynomial fuzzy model-based control systems. An output-feedback polynomial fuzzy controller is employed to drive the system outputs to reach a desired level. SOS-based stability conditions for the three cases (perfectly, partially and imperfectly matched premises) are obtained, which are facilitated by considering different information of membership functions, to determine the system stability and synthesize the controller. With the support of Barbalat's Lemma, it is guaranteed that a stable output-feedback polynomial fuzzy controller will produce no steady state error.
- Chapter 7 considers a tracking problem for polynomial fuzzy model-based control systems. An output-feedback polynomial fuzzy controller is employed to drive the system outputs to follow a reference trajectory. SOS-based stability conditions are obtained to determine the system stability and synthesize the controller where the tracking performance satisfies an H_∞ performance index governing the tracking error.
- Chapter 8 considers a sampled data output-feedback polynomial fuzzy model-based control system which is formed by a nonlinear plant represented by the polynomial fuzzy model and a sampled data output-feedback polynomial fuzzy controller connected in a closed loop. SOS-based stability analysis considering the effect due to sampling and zero-order-hold activities is performed using the input-delay method. SOS-based stability conditions are obtained to determine the system stability and synthesize the controller.

Part IV Advanced Lyapunov Functions

- Chapter 9 proposes a switching polynomial Lyapunov function candidate, which consists of a number of local sub-Lyapunov function candidates, for the stability analysis of polynomial fuzzy model-based control systems where switching is dependent on the system states. When the system state vector falls into the pre-defined local operating domain, the corresponding local sub-Lyapunov function candidate is employed to take care of the system stability. Corresponding to each local sub-Lyapunov function candidate, a local polynomial fuzzy controller is employed for the control of the nonlinear plant resulting in a switching polynomial fuzzy control strategy. A favorable form of switching polynomial Lyapunov function candidate is proposed to make sure

that smooth transition among the local sub-Lyapunov function candidates takes place at the switching boundary for a valid Lyapunov function candidate. SOS-based stability conditions are obtained to determine the system stability and synthesize the controller.

- Chapter 10 proposes a fuzzy polynomial Lyapunov function candidate, which consists of a number of local sub-Lyapunov function candidates, for the stability analysis of polynomial fuzzy model-based control systems where the contribution of each local sub-Lyapunov function candidate to the overall fuzzy polynomial Lyapunov function candidate is governed by the membership functions and fuzzy rules. Piecewise linear membership functions are proposed for the implementation of membership functions in the fuzzy polynomial Lyapunov function candidate to alleviate the difficulty in the stability analysis caused by the time derivative of the membership functions. Furthermore, the piecewise linear membership functions divide the overall operating domain into operating sub-domains. A local polynomial fuzzy controller is proposed for the corresponding operating sub-domains. During the control process, the corresponding local polynomial fuzzy controller is employed for the control of the nonlinear plant resulting in switching control strategy. SOS-based stability conditions are obtained to determine the system stability and synthesize the controller.

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London

Hak-Keung Lam

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Acronyms

FLS	Fuzzy Logic System
FMB	Fuzzy Model-Based
FPLF	Fuzzy Polynomial Lyapunov Function
IT2	Interval Type-2
LMI	Linear Matrix Inequality
MFD	Membership Function-Dependent
MFI	Membership Function-Independent
MRAC	Model Reference Adaptive Control
PDC	Parallel Distributed Compensation
PFMB	Polynomial Fuzzy Model-Based
PLF	Polynomial Lyapunov Function
PLMF	Piecewise Linear Membership Function
PMF	Piecewise Membership Function
SDOF	Sampled Data Output-Feedback
SDP	Semi-Definite Program
SMF	Staircase Membership Function
SOS	Sum of Squares
SPLF	Switching Polynomial Lyapunov Function
T-S	Takagi-Sugeno
T-S-K	Takagi-Sugeno-Kang
TSMF	Taylor Series Membership Function
ZOH	Zero-Order-Hold