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Quantum Field Theory II

Introductions to Quantum Gravity, Supersymmetry and String Theory

Springer

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Preface to Volume II

My motivation in writing this second volume was to have a rather introductory book on quantum gravity,[1] supersymmetry,[2] and string theory[3] for a reader who has had some training in conventional quantum field theory (QFT) dealing with its foundations, with abelian and non-abelian gauge theories including grand unification, and with the basics of renormalization theory as already covered in Vol. I *Quantum Field Theory I: Foundations and Abelian and Non-Abelian Gauge Theories*. This volume is partly based on lectures given to graduate students in theoretical and experimental physics, at an introductory level, emphasizing those parts which are reasonably well understood and for which satisfactory theoretical descriptions have been given.

Quantum gravity is a vast subject,[4] and I obviously have to make a choice in this introductory treatment of the subject. As an introduction, I restrict the study to two different approaches to quantum gravity: the perturbative quantum general relativity approach as the main focus and a non-perturbative background-independent one referred to as "loop quantum gravity" (LQG), where space emerges from the theory itself and is quantized. In LQG we encounter a *QFT in a three-dimensional space*.

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[1]For more advanced books on quantum gravity that I am familiar with, see the following: C. Kiefer (2012): Quantum Gravity, by Oxford University Press, T. Thiemann (2007): Modern Canonical Quantum Gravity, C. Rovelli (2007): Quantum Gravity, as well as of the collection of research investigations in D. Oriti (2009): Approaches to Quantum Gravity, by Cambrige University Press.

[2]For more advanced books on supersymmetry that I am familiar with, see the following books: H. Baer & X. Tata (2006): Weak scale supersymmetry: from superfields to scattering events, M. Dine (2007): Supersymmetry and string theory - beyond the stadard model, S. Weinberg (2000): The Quantum theory of fields III: Supersymmetry, by Cambridge University Press, and P. Binetruy (2006): Supersymmetry, experiments and cosmology by Oxford University Press.

[3]For more advanced books on string theory that I am familiar with, see the following books: K. Becker, M. Becker & J. H. Schwarz (2006): String theory and M-theory - a modern approach, M. Dine (2007): Supersymmetry and string theory - beyond the standard model, and J. Polchinski (2005) : Superstring theory I & II by Cambridge University Press.

[4]See the references given above on quantum gravity.

Some unique features of the treatment given are:

- No previous knowledge of general relativity is required, and the necessary geometrical aspects needed are derived afresh.
- The derivation of field equations and of the expression for the propagator of the graviton in the linearized theory is solved with a gauge constraint, and a constraint necessarily implies that not all the components of the gravitational field may be varied independently—a point which is most often neglected in the literature.
- An elementary treatment is given of the so-called Schwinger-DeWitt technique.
- Non-renormalizability aspects of quantum general relativity are discussed as well as of the renormalizability of some higher-order derivative gravitational theories.
- A proof is given of the Euler-Poincaré Characteristic Theorem which is most often omitted in textbooks.
- A uniqueness property of the invariant product of three Riemann tensors is proved which is also most often omitted in textbooks.
- An introductory treatment is provided of "loop quantum gravity" with sufficient details to get the main ideas across and prepare the reader for more advanced studies.

Supersymmetry is admittedly a theory with mathematical beauty. It unites particles of integer and half-integer spins, i.e., with different spins, but with equal masses in symmetry multiplets. Some important aspects in the treatment of the subject are the following:

- A fundamental property of supersymmetric theories is that the supersymmetry charge (supercharge) operator responsible for interchanging bosonic and fermionic degrees of freedom obviously does not commute with angular momentum (spin) due to different spins arising in a given supermultiplet. This commutation relation is explicitly derived which is most often omitted in textbooks.
- The concept of superspace is introduced, as a direct generalization of the Minkowski one, and the basic theory of integration and differentiation in superspace is developed.
- A derivation is given of the so-called Super-Poincaré algebra satisfied by the generators of supersymmetry and spacetime transformations, which involves commutators and anti-commutators[5] and generalizes the Poincaré algebra of spacetime transformations derived in Vol. I.
- The subject of supersymmetric invariance of integration theory in superspace is developed as it is a key ingredient in defining supersymmetric actions and in constructing supersymmetric extensions of various field theories.
- A panorama of superfields is given including that of the *pure* vector superfield, and complete derivations are provided.

[5] Such an algebra is referred to as a graded algebra.

- Once the theory of supersymmetric invariant integration is developed, and superfields are introduced, supersymmetric extensions of basic field theories are constructed, such as that of Maxwell's theory of electrodynamics; a spin 0–spin 1/2 field theory, referred to as the Wess-Zumino supersymmetric theory *with* interactions; the Yang-Mill field theory; and the standard model.
- There are several advantages of a supersymmetric version of a theory over its non-supersymmetric one. For one thing, the ultraviolet divergence problem is much improved in the former in the sense that divergences originating from fermions loops tend, generally, to cancel those divergent contributions originating from bosons due to their different statistics. The couplings in the supersymmetric version of the standard model merge together more precisely at a high energy. Moreover, this occurs at a higher energy than in the non-supersymmetric theory, getting closer to the Planck one at which gravity is expected to be significant. This gives the hope of unifying gravity with the rest of interactions in a quantum setting.
- Spontaneous symmetry breaking is discussed to account for the mass differences observed in nature of particles of bosonic and fermionic types.
- The underlying geometry necessary for incorporating spinors in general relativity is developed to finally and explicitly derive the expression of the action of the full *supergravity* theory.

In string theory, one encounters a *QFT on two-dimensional surfaces* traced by strings in spacetime, referred to as their worldsheets, with remarkable consequences in spacetime itself, albeit in higher dimensions. If conventional field theories are low-energy effective theories of string theory, then this alone justifies introducing this subject to the student. Some important aspects of the treatment of the subject are the following:

- In string theory, particles that are needed in elementary particle physics arise naturally in the mass spectra of oscillating strings and are not, a priori, assumed to exist or put in by hand in the underlying theory. One of such particles emerging from closed strings is the evasive graviton.
- With the strings being of finite extensions, string theory may, perhaps, provide a better approach than conventional field theory since the latter involves products of distributions at the same spacetime points which are generally ill defined.
- Details are given of all the massless fields in bosonic and superstring theories, including the determination of their inherited degrees of freedom.
- The derived degrees of freedom associated with a massless field in D-dimensional spacetime, together with the eigenvalue equation associated with the mass squared operator associated with such a given massless field, are consistently used to determine the underlying spacetime dimensions D of the bosonic and superstring theories.
- Elements of space compactifications are introduced.
- The basics of the underlying theory of vertices, interactions, and scattering of strings are developed.
- Einstein's theory of gravitation is readily obtained from string theory.
- The Yang-Mills field theory is readily obtained from string theory.

This volume is organized as follows. In Chap. 1, the reader is introduced to quantum gravity, where no previous knowledge of general relativity (GR) is required. All the necessary geometrical aspects are derived afresh leading to explicit general Lagrangians for gravity, including that of GR. The quantum aspect of gravitation, as described by the graviton, is introduced, and perturbative quantum GR is discussed. The so-called Schwinger-DeWitt formalism is developed to compute the one-loop contribution to the theory, and renormalizability aspects of the perturbative theory are also discussed. This follows by introducing the very basics of a non-perturbative, background-independent formulation of quantum gravity, referred to as "loop quantum gravity" which gives rise to a quantization of space and should be interesting to the reader. In Chap. 2, we introduce the reader to supersymmetry and its consequences. In particular, quite a detailed representation is given for the generation of superfields, and the underlying section should provide a useful source of information on superfields. Supersymmetric extensions of Maxwell's theory, as well as of Yang-Mills field theory, and of the standard model are worked out, as mentioned earlier. Spontaneous symmetry breaking, and improvement of the divergence problem in supersymmetric field theory are also covered. The unification of the fundamental couplings in a supersymmetric version of the standard model[6] is then studied. Geometrical aspects necessary to study supergravity are established culminating in the derivation of the full action of the theory. In the final chapter, the reader is introduced to string theory, involving both bosonic and superstrings, and to the analysis of the spectra of the mass (squared) operator associated with the oscillating strings. The properties of the underlying fields, associated with massless particles, encountered in string theory are studied in some detail. Elements of compactification, duality, and D-branes are given, as well as of the generation of vertices and interactions of strings. In the final sections on string theory, we will see how one may recover general relativity and the Yang-Mills field theory from string theory. We have also included two appendices at the end of this volume containing useful information relevant to the rest of this volume and should be consulted by the reader. The problems given at the end of the chapters form an integral part of the books, and many developments in the text depend on the problems and may include, in turn, additional material. They should be attempted by every serious student. *Solutions to all the problems are given* right at the end of the book for the convenience of the reader. We make it a point *pedagogically* to derive things in detail, and some of such details are sometimes *relegated* to appendices at the end of the respective chapters, or worked out in the problems, with the main results *given* in the chapters in question. The very detailed introduction to QFT since its birth in 1926 in Vol. I,[7] as well as the introductions to the chapters, provide the motivations

[6]The standard model consists of the electroweak and QCD theories combined, with a priori underlying symmetry represented by the group products $SU(2) \times U(1) \times SU(3)$.

[7]*Quantum Field Theory I: Foundations and Abelian and Non-Abelian Gauge Theories*. I strongly suggest that the reader goes through the introductory chapter of Vol. I to obtain an overall view of QFT.

and the pedagogical means to handle the technicalities that follow them in these studies.

This volume is suitable as a textbook. Its content may be covered in a 1 year (two semesters) course. Short introductory seminar courses may be also given on quantum gravity, supersymmetry, and string theory.

I often meet students who have a background in conventional quantum field theory mentioned earlier and want to learn about quantum gravity, supersymmetry and string theory but have difficulty in reading more advanced books on these subjects. I thus felt a pedagogical book is needed which puts these topics together and develops them in a coherent introductory and unified manner with a consistent notation which should be useful for the student who wants to learn the underlying different approaches in a more efficient way. He or she may then consult more advanced specialized books, also mentioned earlier, for additional details and further developments, hopefully, with not much difficulty.

I firmly believe that different approaches taken in describing fundamental physics at very high energies or at very small distances should be encouraged and considered as future experiments may confirm directly, or even indirectly, their relevance to the real world.

I hope this book will be useful for a wide range of readers. In particular, I hope that physics graduate students, not only in quantum field theory and high-energy physics but also in other areas of specializations, will also benefit from it as, according to my experience, they seem to have been left out of this fundamental area of physics, as well as instructors and researchers in theoretical physics.

Edouard B. Manoukian

Contents

Notation

- Latin indices $i, j, k, \ldots$ are generally taken to run over 1,2,3, while the Greek indices $\mu, \nu, \ldots$ over $0, 1, 2, 3$ in 4D. Variations do occur when there are many different types of indices to be used, and the meanings should be evident from the presentations.
- The Minkowski metric $\eta_{\mu\nu}$ is defined by $[\eta_{\mu\nu}] = \text{diag}[-1, 1, 1, 1] = [\eta^{\mu\nu}]$ in 4D.
- The charge conjugation matrix is defined by $\mathscr{C} = \mathrm{i}\gamma^2\gamma^0$.
- Unless otherwise stated, the fundamental constants $\hbar$, c are set equal to one.
- The gamma matrices satisfy the anti-commutation relations $\{\gamma^\mu, \gamma^\nu\} = -2\,\eta^{\mu\nu}$.
- $\overline{\psi} = \psi^\dagger\gamma^0$, $\overline{u} = u^\dagger\gamma^0$, $\overline{v} = v^\dagger\gamma^0$. A Hermitian conjugate of a matrix M is denoted by $M^\dagger$, while its complex conjugate is denoted by M^*.
- The Dirac, the Majorana, and the chiral representations of the γ^μ matrices are defined in Appendix I at the end of this volume.
- γ^μ matrices are defined in other dimensions in Appendix I as well.
- The step function is denoted by $\theta(x)$ which is equal to 1 for $x > 0$ and 0 for $x < 0$.