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Fluid and Thermodynamics

Volume 2: Advanced Fluid Mechanics and Thermodynamic Fundamentals

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Preface

Fluid and thermodynamics (FTD) are generally taught at technical universities as separate subjects and this separation can be justified simply by reasons of the assigned time; the elements of each subject can be introduced within a semester of $\sim$15 weeks. Most likely, these outer educational boundaries may even have well furthered this separation. Intellectually, the two subjects, however, belong together, especially since for all but ideal fluids the second law of thermodynamics imposes constraint conditions on the parameters of the governing equations (generally partial differential equations) that are then used in the fluid dynamic part of the joint effort to construct solutions to physically motivated initial boundary value problems that teach us important facts of the behavior of the motion of the fluid under certain circumstances.

One of the authors (K.H.) found this combination of fluid and thermodynamics as an assigned one-semester course, when he started in 1987 in the Department of Mechanics at Technische Universität Darmstadt (at that time 'Technische Hochschule') as successor of the late Prof. Dr. rer.nat. Ernst Becker (1929–1984). With K.H's emphasized interest in continuum mechanics and thermodynamics, this dual understanding of the mathematical description of fluid matter was ideal and the assignment to teach it was a welcome challenge, which was declared as a 'credo' to the working environment in both teaching and research in his group.

The course notes of FTD taught to upper-class electrical engineers for 18 years were quickly worked out into the book 'Fluid und Thermodynamik – eine Einführung' and published by Springer Verlag, Berlin etc., (ISBN 3-540-59235-0, second edition). All the chapters of this book—some slightly extended—have been translated (by K.H.) into the English language and are interwoven in this treatise with chapters, which, as a whole, should provide a fairly detailed understanding of FTD.

All subjects of this treatise of FTD have been taught in one or another form as lectures in courses to students at Technische Universität Darmstadt, Swiss Federal Institute of Technology in Zürich (ETHZ), and in guest lectures in advanced courses at other universities and research institutions worldwide. The audience in these courses consisted of students, doctoral candidates and postdoctoral assistants of engineering (civil, mechanical, chemical, mechanics), natural sciences (meteorologists, oceanographers, geophysicists), mathematics, and physics. Some of the topics included are as follows:

- Fluid mechanics,
- Continuum mechanics and thermodynamics,
- Mechanics of environmentally related systems (glacier, ice-sheet mechanics, physical oceanography, lake physics, soil motion, avalanches, debris, and mud flows),
- Vorticity and angular momentum,
- Turbulence modeling (of zeroth, first and second order),
- Regular and singular perturbations,
- Continuum mechanics and thermodynamics of mixtures,
- Continuum mechanics and thermodynamics of COSSERAT continua and COSSERAT mixtures,
- Theoretical glaciology,
- Shallow creeping flows of landslides, glaciers, and ice sheets,

and others. It is hoped that we were successful in designing a coherent picture of the intended text FTD.

Writing the book chapters also profited from books that were written earlier by us and co-authors [1–6].

Fluid and Thermodynamics
Volume 1: Basic Fluid Mechanics

This volume consists of 10 chapters and begins in an introductory **Chap. 1** with some historical facts, definition of the subject field and lists the most important properties of liquids.

This descriptive account is then followed in **Chap. 2** by the simple mathematical description of the fundamental hydrostatic equation and its use in analyses of equilibrium of fluid systems and stability of floating bodies, the derivation of the ARCHIMEDEAN principle and determination of the pressure distribution in the atmosphere.

Chapter 3 deals with hydrodynamics of ideal incompressible (density preserving) fluids. Streamlines, trajectories, and streaklines are defined. A careful derivation of the balances of mass and linear momentum is given and it is shown how the BERNOULLI equation is derived from the balance law of momentum and how it is used in applications. In one-dimensional smooth flow problems the momentum and BERNOULLI equations are equivalent. For discontinuous processes with jumps this is not so. Nevertheless the BERNOULLI equation is a very useful equation in many engineering applications. This chapter ends with the balance law of moment of momentum and its application for EULER's turbine equation.

The conservation law of angular momentum, presented in **Chap. 4**, provides the occasion to define circulation and vorticity and the vorticity theorems, among them those of Helmholtz and Ertel. The goal of this chapter is to build a fundamental understanding of vorticity.

In **Chap. 5** a collection of simple flow problems in ideal fluids is presented. It is shown how vector analytical methods are used to demonstrate the differential geometric properties of vortex-free flow fields and to evaluate the motion-induced force on a body in a potential field. The concept of virtual mass is defined and two-dimensional fluid potential flow is outlined.

This almanac of flows of ideal fluids is complemented in **Chap. 6** by the presentation of the solution techniques of two-dimensional potential flow by complex-valued function theoretical methods using conformal mappings. Potential flows around two-dimensional air foils, laminar free jets, and the Schwarz–Chrystoffel transformations are employed to construct the mathematical descriptions of such flows through a slit or several slits, around air wings, free jets, and in ducts bounding an ideal fluid.

The mathematical physical study of viscous flows starts in **Chap. 7** with the derivation of the general stress–strain rate relation of viscous fluids, in particular Navier–Stokes fluids and more generally, non-Newtonian fluids. Application of these equations to viscometric flows, liquid films, Poiseuille flow, and the slide bearing theory due to Reynolds and Sommerfeld demonstrate their use in an engineering context. Creeping flow for a pseudo-plastic fluid with free surface then shows the application in the glaciological-geological context.

Chapter 8 continues with the study of two-dimensional and three-dimensional simple flow of the Navier–Stokes equations. Hagen–Poiseuille flow and the Ekman theory of the wall-near wall-parallel flow on a rotating frame (Earth) and its generalization are presented as solutions of the Navier–Stokes equations in the half-space above an oscillating wall and that of a stationary axisymmetric laminar jet. This then leads to the presentation of Prandtl's boundary layer theory with flows around wedges and the Blasius boundary layer and others.

In **Chap. 9** two- and three-dimensional boundary layer flows in the vicinity of a stagnation point are studied as are flows around wedges and along wedge sidewalls. The flow, induced in the half plane above a rotating plane, is also determined. The technique of the boundary layer approach is commenced with the Blasius flow, but more importantly, the boundary layer solution technique for the Navier–Stokes equations is explained by use of the method of matched asymptotic expansions. Moreover, the global laws of the steady boundary layer theory are explained with the aid of the Holstein–Bohlen procedure. The chapter ends with a brief study of non-stationary boundary layers, in which an impulsive start from rest, flow in the vicinity of a pulsating body, oscillation induced drift current, and non-stationary plate boundary layers are studied.

In **Chap. 10** pipe flow is studied for laminar (Hagen–Poiseuille) as well as for turbulent flows; this situation culminates via a dimensional analysis to the well-known Moody diagram. The volume ends in this chapter with the plane boundary layer flow along a wall due to Prandtl and von Kármán with the famous logarithmic velocity profile. This last problem is later reanalyzed as the controversies between a power and logarithmic velocity profile near walls is still ongoing research today.

Fluid and Thermodynamics
Volume 2: Advanced Fluid Mechanics and Thermodynamic Fundamentals

This volume consists of 10 chapters and commences in **Chap. 11** with the determination of the creeping motion around spheres at rest in a NEWTONian fluid. This is a classical problem of singular perturbations in the form of matched asymptotic expansions. For creeping flow the acceleration terms in NEWTON's law can be ignored to approximately calculate flow around the sphere by this so-called STOKES approximation. It turns out that far away from the sphere the acceleration terms become larger than those in the STOKES solution, so that the latter solution violates the boundary conditions at infinity. This lowest order correction of the flow around the sphere is due to OSEEN (1910). In a systematic perturbation expansion the outer —OSEEN—series and the inner—STOKES—series with the small REYNOLDS number as perturbation parameter must be matched together to determine all boundary and transition conditions of inner and outer expansions. This procedure is rather tricky, i.e., not easy to understand for beginners. This theory, originally due KAPLUN and to LAGERSTRÖM has been extended, and the drag coefficient for the sphere, which also can be measured is expressible in terms of a series expansion of powers of the REYNOLDS number. However, for REYNOLDS numbers larger than unity, convergence to measured values is poor. About 20–30 years ago a new mathematical approach was designed—the so-called Homotopy Analysis Method; it is based on an entirely different expansion technique, and results for the drag coefficient lie much closer to the experimental values than values obtained with the 'classical' matched asymptotic expansion, as shown in Fig. 11.11. Incidentally the laminar flow of a viscous fluid around a cylinder can analogously be treated, but is not contained in this treatise.

Chapter 12 is devoted to the approximate determination of the velocity field in a shallow layer of ice or granular soil, treated as a non-NEWTONian material flowing under the action of its own weight and assuming its velocity to be so small that STOKES flow can be assumed. Two limiting cases can be analyzed: (i) In the first, the flowing material on a steep slope (which is the case for creeping landslides or snow on mountain topographies with inclination angles that are large). (ii) In the second case the inclination angles are small. Situation (ii) is apt to ice flow in large ice sheets such as Greenland and Antarctica, important in climate scenarios in a warming atmosphere. We derive perturbation schemes in terms of a shallowness parameter in the two situations and discuss applications under real-world conditions.

In shallow rapid gravity driven free surface flows the acceleration terms in NEWTON's law are no longer negligible. **Chapter 13** is devoted to such granular flows in an attempt to introduce the reader to the challenging theory of the dynamical behavior of fluidized cohesionless granular materials in avalanches of snow, debris and mud, etc. The theoretical description of moving layers of granular assemblies begins with the one-dimensional depth integrated MOHR–COULOMB plastic layer flows down inclines—the so-called SAVAGE–HUTTER theory, but then continues with the general formulation of the model equations referred to topography following curvilinear coordinates with all its peculiarities in the theory and the use of shock-capturing numerical integration techniques.

Chapter 14 on uniqueness and stability provides a first flavor into the subject of laminar-turbulent transition. Two different theoretical concepts are in use and both assume that the laminar–turbulent transition is a question of loss of stability of the laminar motion. With the use of the energy method one tries to find upper bound conditions for the laminar flow to be stable. More successful for pinpointing the laminar-turbulent transition has been the method of linear instability analysis, in which a lowest bound is searched for, at which the onset of deviations from the laminar flow is taking place.

In **Chap. 15**, a detailed introduction to the modeling of turbulence is given. Filter operations are introduced to separate the physical balance laws into evolution equations for the averaged fields on the one hand, and into fluctuating or pulsating fields on the other hand. This procedure generates averages of products of fluctuating quantities, for which closure relations must be formulated. Depending upon the complexity of these closure relations, so-called zeroth, first, and higher order turbulence models are obtained: simple algebraic gradient-type relations for the flux terms, one or two equation models, e.g., k-ε, k-ω, in which evolution equations for the averaged correlation products are formulated, etc. This is done for density preserving fluids as well as so-called BOUSSINESQ fluids and convection fluids on a rotating frame (Earth), which are important models to describe atmospheric and oceanic flows.

Chapter 16 goes back one step by scrutinizing the early zeroth order closure relations as proposed by PRANDTL, VON KÁRMÁN and collaborators. The basis is BOSSINESQ's (1872) ansatz for the shear stress in plane parallel flow, τ_{12}, which is expressed to be proportional to the corresponding averaged shear rate $\partial\bar{v}_1/\partial x_2$ with coefficient of proportionality $\rho\varepsilon$, where ρ is the density and ε a kinematic turbulent viscosity or turbulent diffusivity [$\mathrm{m}^2\ \mathrm{s}^{-1}$]. In turbulence theory the flux terms of momentum, heat, and suspended mass are all parameterized as gradient-type relations with turbulent diffusivities treated as constants. PRANDTL realized from data collected in his institute that ε was not a constant but depended on his mixing length squared and the magnitude of the shear rate (PRANDTL 1925). This proposal was later improved (PRANDTL 1942) to amend the unsatisfactory agreement at positions where shear rates disappeared. The 1942-law is still local, which means that the REYNOLDS stress tensor at a spatial point depends on spatial velocity derivatives at the *same* position. PRANDTL in a second proposal of his 1942-paper suggested that the turbulent diffusivity should depend on the velocity *difference* at the points where the velocity of the turbulent path assumes maximum and minimum values. This proposal introduces some non-locality, yielded better agreement with data, but

PRANDTL left the gradient-type dependence in order to stay in conformity with BOUSSINESQ. It does neither become apparent nor clear that PRANDTL or the modelers at that time would have realized that non-local effects would be the cause for better agreement of the theoretical formulations with data. The proposal of complete non-local behavior of the REYNOLDS stress parameterization came in 1991 by P. EGOLF and subsequent research articles during $\sim$20 years, in which also the local strain rate (= local velocity gradient) is replaced by a difference quotient. We motivate and explain the proposed Difference Quotient Turbulence Model (DQTM) and demonstrate that for standard two-dimensional configurations analyzed in this chapter its performance is superior to other zeroth order models.

The next two chapters are devoted to thermodynamics; first, fundamentals are attacked and, second a field formulation is presented and explored.

Class experience has taught us that thermodynamic fundamentals (**Chap. 17**) are difficult to understand for novel readers. Utmost caution is therefore exercised to precisely introduce terminology such as 'states', 'processes', 'extensive', 'intensive', and 'molar state variables' as well as concepts like 'adiabatic', and 'diathermal walls', 'empirical' and 'absolute temperature', 'equations of state', and 'reversible' and 'irreversible processes'. The core of this chapter is, however, the presentation of the First and Second Law of Thermodynamics. The *first law* balances the energies. It states that the time rate of change of the kinetic plus internal energies are balanced by the mechanical power of the stresses and the body forces plus the thermal analogies, which are the flux of heat through the boundary plus the specific radiation also referred to as energy supply. This conservation law then leads to the definitions of the caloric equations of state and the definitions of specific heats. The Second Law of Thermodynamics is likely the most difficult to understand and it is introduced here as a balance law for the entropy and states that all physical processes are irreversible. We motivate this law by going from easy and simple systems to more complex systems by generalization and culminate in this tour with the Second Law as the statement that entropy production rate cannot be negative. Examples illustrate the implications in simple physical systems and show where the two variants of entropy principles may lead to different answers.

Chapter 18 extends and applies the above concepts to continuous material systems. The Second Law is written in global form as a balance law of entropy with flux, supply and production quantities, which can be written in local form as a differential statement. The particular form of the Second Law then depends upon which postulates the individual terms in the entropy balance are subjected to. When the entropy flux equals heat flux divided by absolute temperature and the entropy production rate density is requested to be non-negative, the entropy balance law appears as the CLAUSIUS–DUHEM inequality and its exploitation follows the axiomatic procedure of open systems thermodynamics as introduced by COLEMAN and NOLL. When the entropy flux is left arbitrary but is of the same function class as the other constitutive relations and the entropy supply rate density is identically zero, then the entropy inequality appears in the form of MÜLLER. In both cases the Second Law is expressed by the requirement that the entropy production rate density must be non-negative, but details of the exploitation of the Second Law in the two cases are subtly different from one another. For standard media such as elastic and/or viscous fluids the results are the same. However, for complex media they may well differ from one another. Examples will illustrate the procedures and results.

Chapter 19 on gas dynamics illustrates a technically important example of a fluid field theory, where the information deduced by the Second Law of Thermodynamics delivers important properties, expressed by the thermal and caloric equations of state of, say, ideal and real gases. We briefly touch problems of acoustics, steady isentropic flow processes and their stream filament theory. The description of the propagation of small perturbations in a gas serves in its one-dimensional form ideally as a model for the propagation of sound, for e.g. in a flute or organ pipe, and it can be used to explain the Doppler shift occurring when the sound source is moving relative to the receiver. Moreover, with the stream filament theory the sub- and supersonic flow through a nozzle can be explained. In a final section the three-dimensional theory of shocks is derived as the set of jump conditions on surfaces for the balance laws of mass, momentum, energy, and entropy. Their exploitation is illustrated for steady surfaces for simple fluids under adiabatic flow conditions. These problems are classics; gas dynamics, indeed forms an important advanced technical field that was developed in the twentieth century as a subject of aerodynamics and astronautics and important specialties of mechanical engineering.

Chapter 20 is devoted to the subjects 'Dimensional analysis, similitude and physical experimentation at laboratory scale', topics often not systematically taught at higher technical education. However, no insider would deny their usefulness. Books treating these subjects separately and in sufficient detail have appeared since the mid-twentieth century. We give an account of dimensional analysis, define dimensional homogeneity of functions of mathematical physics, the properties of which culminate in Buckingham's theorem (which is proved in an appendix to the chapter); its use is illustrated by a diversity of problems from general fluid dynamics, gas dynamics, and thermal sciences, e.g., propagation of a shock from a point source, rising gas bubbles, Rayleigh–Bénard instability, etc. The theory of physical models develops rules, how to down- or up-scale physical processes from the size of a prototype to the size of the model. The theory shows that in general such scaling transformations are practically never exactly possible, so that scale effects enter in these cases, which distort the model results in comparison to those in the prototype. In hydraulic applications, this leads to the so-called Froude and Reynolds models, in which either the Froude or Reynolds number, respectively, remains a mapping invariant but not the other. Application on sediment transport in rivers, heat transfer in forced convection, etc., illustrate the difficulties. The chapter ends with the characterization of dimensional homogeneity of the equations describing physical processes by their governing differential equations. The Navier–Stokes–Fourier–Fick fluid equations serve as illustration.

The intention of this treatise is, apart from presenting its addressed subjects, a clear, detailed, and somewhat rigorous mathematical presentation of FTD on the basis of limited knowledge as a prerequisite. Calculus or analysis of functions of a single or several variables, linear algebra and the basics of ordinary and partial differential equations are assumed to be known, as is the theory of complex functions. The latter is not universally taught in engineering curricula of universities; we believe that readers not equipped with the theory of complex functions can easily familiarize themselves with its basics in a few weeks' reading effort.

A second goal of this treatise is to frame the individual subjects in their historical content by providing biographical sketches of the inventors of the particular concepts. The science of fluid and thermodynamics began in the Western world more than 2000 years ago, e.g., by ARCHIMEDES in Syracuse. First careful observations on turbulence were described by LEONARDO DA VINCI and on the motion of falling bodies by GALILEO GALILEI. Mathematical description of the motion of physical bodies was begun by ISAAC NEWTON, and DESCARTES. EULER and father JOHANN and son DANIEL BERNOULLI introduced, among others, the continuous methods for ideal, i.e., reversible materials. Most of this research took place in the seventeenth and eighteenth centuries and was perfected in the upcoming nineteenth and twentieth centuries. The recognition of the energy balance equation and the entropy imbalance statement as physical laws are achievements of the nineteenth and first part of the twentieth centuries and are associated with scientists like SADI CARNOT, JULIUS MAYER, HERMANN HELMHOLTZ, RUDOLF CLAUSIUS, PIERRE MAURICE MARIE DUHEM, WILLIAM THOMSON (LORD KELVIN), WILLIARD GIBBS, and MAX PLANK, to name a few.

The solutions of the (initial) boundary value problems which ensue from the emerging equations have been solved by a large number of follow-up scientists from the mid-nineteenth century to present, of whom a few stand out distinguishingly: OSBORNE REYNOLDS, LORD RAYLEIGH, LUDWIG PRANDTL, THEODORE VON KÁRMÁN, G.I. TAYLOR, HERMANN SCHLICHTING, and many others. The history, which evolved from the work of all these scientists, is fascinating. By listing short biographical sketches of those scientists who contributed to the development of fluid and thermodynamics, we hope to guide the reader to a coherent historical development of the fascinating subject of fluid and thermodynamics.

We regard this dual approach as a justified procedure, especially since the twenty-first century university students do no longer sufficiently appreciate the fact, on which shoulders of giants and predecessors we stand.

The books have been jointly drafted by us from notes that accumulated during years. As mentioned before, the Chaps. 1–3, 5, 7, 10, 17–20 are translated (and partly revised) from 'Fluid- und Thermodynamik – eine Einführung'. Many of the other chapters were composed in handwriting and TEXed by K.H. and substantially improved and polished by Y.W. We share equal responsibility for the content and the errors that still remain. Figures, which are taken from others, are reproduced and mostly redrawn, but mentioned in the acknowledgment and/or figure captions. Nevertheless a substantial number of figures have been designed by us. However, we received help for their electronic production: Mr. ANDREAS ROHRER, from the Laboratory of Hydraulics, Hydrology and Glaciology at ETH Zurich (VAW), drew figures for Chaps. 8 and 9 and the student assistants Mr. WALDEMAR SURNIN and Mr. JAN BATTRAM from the Institute of Fluid Dynamics at Technische Universität Darmstadt aided in the production of figures of several other chapters. Mr. ANDREAS

Schlumpf from VAW and Ms. Alexandra Paunica and Prof. Ioana Luca drew figures for Chap. 6 and several other chapters.

It is custom of most publishers to ask referees to review book manuscripts shortly before submission for printing by experts of the subjects treated in the forthcoming book. It is, however, also almost consequential that reviewers for a two-volume treatise of more than 1200 pages can hardly be found, simply because of the excessive labor that goes with such an assignment. Nevertheless this burden was taken up by two emeriti, Dr.-Ing. Peter Haupt, Professor of Mechanics at the University of Kassel, Germany and Dr. rer. nat, Dr. h.c. Hans Dieter Alber, Professor of Mathematics, Technical University, Darmstadt, Germany. We thoroughly thank these colleagues for their extensive help. Their criticisms and recommendations are gratefully incorporated in the final version of the manuscript.

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This treatise was planned as a three-volume project, and, indeed, two chapters of a possible volume III have already been written. We still hold up this intention, but the advanced age of one of us does not guarantee that we will be successful in this endeavor. We shall see …

Finally, we thank Springer Verlag, and in particular Dr. Annett Buettner, for the interest in our FTD treatise and AGEM^2, in general.

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