

SpringerBriefs in Mathematical Physics

Volume 12

Series editors

Nathanaël Berestycki, Cambridge, UK

Mihalis Dafermos, Cambridge, UK

Tohru Eguchi, Tokyo, Japan

Atsuo Kuniba, Tokyo, Japan

Matilde Marcolli, Pasadena, USA

Bruno Nachtergaele, Davis, USA

More information about this series at <http://www.springer.com/series/11953>

Gioia Carinci · Anna De Masi
Cristian Giardinà · Errico Presutti

Free Boundary Problems in PDEs and Particle Systems

 Springer

Gioia Carinci
Delft University of Technology
Delft
The Netherlands

Anna De Masi
Dipartimento di Matematica
Universita di L'Aquila
L'Aquila
Italy

Cristian Giardinà
Dipartimento di Matematica
Università di Modena e Reggio Emilia
Modena
Italy

Errico Presutti
Gran Sasso Science Institute
L'Aquila
Italy

ISSN 2197-1757 ISSN 2197-1765 (electronic)
SpringerBriefs in Mathematical Physics
ISBN 978-3-319-33369-4 ISBN 978-3-319-33370-0 (eBook)
DOI 10.1007/978-3-319-33370-0

Library of Congress Control Number: 2016940811

© The Author(s) 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

Contents

1	Introduction	1
Part I The Basic Model		
2	Introduction to Part I	7
	References	10
3	The Basic Model, Definitions and Results	11
	3.1 The Basic Problem	11
	3.2 Stationary Solutions	12
	3.3 The FBP for the Basic Model	13
	3.4 Main Theorem: Existence and Uniqueness	14
	3.5 The Upper and Lower Barriers	15
	3.6 Mass Transport	17
	3.7 Barrier Theorems	18
	References	20
4	Regularity Properties of the Barriers	21
	4.1 Equi-Boundedness	21
	4.2 Space Equi-Continuity	22
	4.3 Time Equi-Continuity	24
5	Lipschitz and L^1 Estimates	27
	5.1 Elementary Inequalities	27
	5.2 Lipschitz Properties	28
	5.3 L^1 Estimates	29
6	Mass Transport Inequalities	31
	6.1 Partial Order and Mass Transport	31
	6.2 A Relaxed Notion of Partial Order	32
	6.3 Inequalities for the Cut and the Free Evolution Operators	33
	6.4 Inequalities for the Barriers	36

7	The Limit Theorems on Barriers	41
7.1	The Limit Function ψ	41
7.1.1	Convergence of the Upper Barriers.	41
7.1.2	Independence of τ	43
7.1.3	Continuity at 0	44
7.2	Proof of Theorem 3.14	45
	Reference	45
8	Brownian Motion and the Heat Equation	47
8.1	Brownian Motion on the Line	47
8.2	Reflected Brownian Motion with Mass Injection	48
8.3	Brownian Motion with Reflection at 0 and Absorption at the Edge	49
8.4	Mass Lost at the Edge	51
	References	53
9	Existence of Optimal Sequences	55
9.1	The Existence Theorem	55
9.2	The First Step of the Iteration	56
9.3	The Iteration	59
	Reference	59
10	Proof of the Main Theorem	61
10.1	The Key Inequality	61
10.2	Proof of Theorem 3.15	61
10.3	Proof of Theorem 10.1	62
10.3.1	The First Step of the Induction	62
10.3.2	A Stochastic Inequality	65
10.3.3	The Generic Step of the Induction	68
10.4	Proof of Theorem 3.2	69
	Reference	69
11	The Basic Particle Model and Its Hydrodynamic Limit	71
11.1	The Model and the Main Result.	71
11.2	Strategy of Proof	72
11.3	The Stochastic Barriers.	73
11.3.1	Stochastic Inequalities: Lower Bound	74
11.3.2	Stochastic Inequalities: Upper Bound	75
11.4	Hydrodynamic Limit for the Stochastic Barriers.	76
11.4.1	Semi-norms.	76
11.4.2	The Key Estimate	79
11.5	Proof of Theorem 11.1	83
Part II Variants of the Basic Model		
12	Introduction to Part II	87
	References	88

- 13 Independent Walkers with Current Reservoirs** 89
 - 13.1 Introduction. 89
 - 13.2 Definition of the Model 91
 - 13.3 Hydrodynamic Limit 92
 - References 95

- 14 Beyond Diffusive Scaling** 97
 - 14.1 Introduction. 97
 - 14.2 Stationary Density Profiles 98
 - 14.3 The Law of the Total Mass. 98
 - 14.4 Super-Hydrodynamic Limit 99
 - References 100

- 15 Other Models** 101
 - 15.1 Cells Evolution in an Active Environment. 102
 - 15.2 The Brunet-Derrida Evolution-Selection Mechanism. 103
 - 15.3 The Durrett and Remenik Model 104
 - 15.4 Models with Two Species. 104
 - 15.5 Interface Models 106
 - References 106

- Index** 109