Topological Fixed Point Theory for Singlevalued and Multivalued Mappings and Applications

Afif Ben Amar • Donal O'Regan

Topological Fixed Point Theory for Singlevalued and Multivalued Mappings and Applications

Springer

Afif Ben Amar
Department of Mathematics
University of Sfax, Faculty of Sciences
Sfax, Tunisia

Donal O'Regan
School of Mathematics
National University of Ireland, Galway
Galway, Ireland

ISBN 978-3-319-31947-6 ISBN 978-3-319-31948-3 (eBook)
DOI 10.1007/978-3-319-31948-3

Library of Congress Control Number: 2016936666

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

To my parents Fathi and Mounira
To my wife Faten and our children Hadil, Hiba, and Youssef
and
To my brothers Imed, Aref, and my sister Alyssa

Afif Ben Amar

To my wife Alice and our children Aoife, Lorna, Daniel, and Niamh

Donal O'Regan

Preface

Fixed point theory is a powerful and fruitful tool in modern mathematics and may be considered as a core subject in nonlinear analysis. In the last 50 years, fixed point theory has been a flourishing area of research. In this book, we introduce topological fixed point theory for several classes of single- and multivalued maps. The selected topics reflect our particular interests.

The text is divided into seven chapters. In Chap. 1, we present basic notions in locally convex topological vector spaces. Special attention is devoted to weak compactness, in particular to the theorems of Eberlein–Šmulian, Grothendieck, and Dunford–Pettis. Leray–Schauder alternatives and eigenvalue problems for decomposable single-valued nonlinear weakly compact operators in Dunford–Pettis spaces are considered in Chap. 2. In Chap. 3, we present some variants of Schauder, Krasnoselskii, Sadovskii, and Leray–Schauder-type fixed point theorems for different classes of weakly sequentially continuous (resp. sequentially continuous) operators on general Banach spaces (resp. locally convex spaces). Sadovskii, Furi–Pera, and Krasnoselskii fixed point theorems and nonlinear Leray–Schauder alternatives in the framework of weak topologies and involving multivalued mappings with weakly sequentially closed graph are considered in Chap. 4. The results are formulated in terms of axiomatic measures of weak noncompactness. In Chap. 5, we present some fixed point theorems in a nonempty closed convex of any Banach algebras or Banach algebras satisfying a sequential condition $(\mathcal{P})$ for the sum and the product of nonlinear weakly sequentially continuous operators. We illustrate the theory by considering functional integral and partial differential equations. The existence of fixed points and nonlinear Leray–Schauder alternatives for different classes of nonlinear (ws)-compact operators (weakly condensing, 1-set weakly contractive, strictly quasi-bounded) defined on an unbounded closed convex subset of a Banach space is discussed in Chap. 6. We also discuss the existence of nonlinear eigenvalues and eigenvectors and surjectivity of quasi-bounded operators. In Chap. 7, we present some approximate fixed point theorems for multivalued mappings defined on Banach spaces. Weak and strong topologies play a role here and both bounded and unbounded regions are considered. A method is developed indicating how to

use approximate fixed point theorems to prove the existence of approximate Nash equilibria for noncooperative games.

We hope the book will be of use to graduate students and theoretical and applied mathematicians who work in fixed point theory, integral equations, ordinary and partial differential equations, game theory, and other related areas.

Sfax, Tunisia — Afif Ben Amar
Galway, Ireland — Donal O'Regan

Contents