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Basics of Thermal Field Theory

A Tutorial on Perturbative Computations

Springer

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Preface

These notes are based on lectures delivered at the Universities of Bielefeld and Helsinki, between 2004 and 2015, as well as at a number of summer and winter schools, between 1996 and 2015. The early sections were strongly influenced by lectures by Keijo Kajantie at the University of Helsinki, in the early 1990s. Obviously, the lectures additionally owe an enormous gratitude to existing textbooks and literature, particularly the classic monograph by Joseph Kapusta.

There are several good textbooks on finite-temperature field theory, and no attempt is made here to join that group. Rather, the goal is to offer an elementary exposition of the basics of perturbative thermal field theory, in an explicit "hands-on" style which can hopefully more or less directly be transported to the classroom. The presentation is meant to be self-contained and display also intermediate steps. The idea is, roughly, that each numbered section could constitute a single lecture. Referencing is sparse; on more advanced topics, as well as on historically accurate references, the reader is advised to consult the textbooks and review articles in Refs. [1–12].

These notes could not have been put together without the helpful influence of many people, varying from students with persistent requests for clarification; colleagues who have used parts of an early version of these notes in their own lectures and shared their experiences with us; colleagues whose interest in specific topics has inspired us to add corresponding material to these notes; alert readers who have informed us about typographic errors and suggested improvements; and collaborators from whom we have learned parts of the material presented here. Let us gratefully acknowledge in particular Gert Aarts, Chris Korthals Altes, Dietrich Bödeker, Yannis Burnier, Stefano Capitani, Simon Caron-Huot, Jacopo Ghiglieri, Ioan Ghisoiu, Keijo Kajantie, Aleksi Kurkela, Harvey Meyer, Guy Moore, Paul Romatschke, Kari Rummukainen, York Schröder, Mikhail Shaposhnikov, Markus Thoma, and Mikko Vepsäläinen.

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Notation

In thermal field theory, both Euclidean and Minkowskian spacetimes play a role.

In the Euclidean case, we write

$$X \equiv (\tau, x^i) \ , \quad x \equiv |\mathbf{x}| \ , \quad S_E = \int_X L_E \ , \tag{1}$$

where $i = 1, \ldots, d$,

$$\int_X \equiv \int_0^\beta \mathrm{d}\tau \int_\mathbf{x} \ , \quad \int_\mathbf{x} \equiv \int \mathrm{d}^d\mathbf{x} \ , \quad \beta \equiv \frac{1}{T} \ , \tag{2}$$

and d is the space dimensionality. Fourier analysis is carried out in the Matsubara formalism via

$$K \equiv (k_n, k_i) \ , \quad k \equiv |\mathbf{k}| \ , \quad \phi(X) = \sum\!\!\!\!\!\!\!\int_K \tilde{\phi}(K)\, e^{iK\cdot X} \ , \tag{3}$$

where

$$\sum\!\!\!\!\!\!\!\int_K \equiv T \sum_{k_n} \int_\mathbf{k} \ , \quad \int_\mathbf{k} \equiv \int \frac{\mathrm{d}^d\mathbf{k}}{(2\pi)^d} \ . \tag{4}$$

Here, k_n stands for discrete Matsubara frequencies, which at times are also denoted by ω_n. In the case of antiperiodic functions, the summation is written as $T\sum_{\{k_n\}}$. The squares of four-vectors read $K^2 = k_n^2 + k^2$ and $X^2 = \tau^2 + x^2$, but the Euclidean scalar product between K and X is defined as

$$K \cdot X = k_n \tau + \sum_{i=1}^{d} k_i x^i = k_n \tau - \mathbf{k} \cdot \mathbf{x} \ , \tag{5}$$

where the vector notation is reserved for contravariant Minkowskian vectors: $\mathbf{x} = (x^i)$, $\mathbf{k} = (k^i)$. If a chemical potential is also present, we denote $\tilde{k}_n \equiv k_n + i\mu$.

In the Minkowskian case, we have

$$\mathcal{X} \equiv (t, \mathbf{x}) \; , \quad x \equiv |\mathbf{x}| \; , \quad \mathcal{S}_M = \int_{\mathcal{X}} \mathcal{L}_M \; , \tag{6}$$

where $\int_{\mathcal{X}} \equiv \int \mathrm{d}x^0 \int_{\mathbf{x}}$. Fourier analysis proceeds via

$$\mathcal{K} \equiv (k^0, \mathbf{k}) \; , \quad k \equiv |\mathbf{k}| \; , \quad \phi(\mathcal{X}) = \int_{\mathcal{K}} \tilde{\phi}(\mathcal{K})\, e^{i\mathcal{K}\cdot\mathcal{X}} \; , \tag{7}$$

where $\int_{\mathcal{K}} = \int \frac{\mathrm{d}k^0}{2\pi} \int_{\mathbf{k}}$, and the metric is chosen to be of the "mostly minus" form,

$$\mathcal{K} \cdot \mathcal{X} = k^0 x^0 - \mathbf{k} \cdot \mathbf{x} \; . \tag{8}$$

No special notation is introduced for the case where a Minkowskian four-vector is on-shell, i.e., when $\mathcal{K} = (E_k, \mathbf{k})$; this is to be understood from the context. The argument of a field ϕ is taken to indicate whether the configuration space is Euclidean or Minkowskian. If not specified otherwise, momentum integrations are regulated by defining the spatial measure in $d = 3 - 2\epsilon$ dimensions, whereas the spacetime dimensionality is denoted by $D = 4 - 2\epsilon$. A Greek index takes values in the set $\{0, \ldots, d\}$ and a Latin one in $\{1, \ldots, d\}$.

Finally, we note that we work consistently in units where the speed of light c and the Boltzmann constant k_B have been set to unity. The reduced Planck constant $\hbar$ also equals unity in most places, excluding the first chapter (on quantum mechanics) as well as some later discussions where we want to emphasize the distinction between quantum and classical descriptions.

General Outline

Physics Context

From the physics point of view, there are two important contexts in which relativistic thermal field theory is being widely applied: cosmology and the theoretical description of heavy ion collision experiments.

In cosmology, the temperatures considered vary hugely, ranging from $T \simeq 10^{15}$ GeV to $T \simeq 10^{-3}$ eV. Contemporary challenges in the field include figuring out explanations for the existence of dark matter, the observed antisymmetry in the amounts of matter and antimatter, and the formation of large-scale structures from small initial density perturbations. (The origin of initial density perturbations itself is generally considered to be a nonthermal problem, associated with an early period of inflation.) An important further issue is that of equilibration, i.e., details of the processes through which the inflationary state turned into a thermal plasma and in particular what the highest temperature reached during this epoch was. It is notable that most of these topics are assumed to be associated with weak or even superweak interactions, whereas strong interactions (QCD) only play a background role. A notable exception to this is light element nucleosynthesis, but this well-studied topic is not in the center of our current focus.

In heavy ion collisions, in contrast, strong interactions do play a major role. The lifetime of the thermal fireball created in such a collision is ~ 10 fm/c, and the maximal temperature reached is in the range of a few hundred MeV. Weak interactions are too slow to take place within the lifetime of the system. Prominent observables are the yields of different particle species, the quenching of energetic jets, and the hydrodynamic properties of the plasma that can be deduced from the observed particle yields. An important issue is again how fast an initial quantum-mechanical state turns into an essentially incoherent thermal plasma.

Despite many differences in the physics questions posed and in the microscopic forces underlying cosmology and heavy ion collision phenomena, there are also similarities. Most importantly, gauge interactions (whether weak or strong) are essential in both contexts. Because of asymptotic freedom, the strong interactions of QCD also become “weak” at sufficiently high temperatures. It is for this reason that many techniques, such as the resummations that are needed for developing a formally consistent weak-coupling expansion, can be applied in both contexts. The topics covered in the present notes have been chosen with both fields of application in mind.

Organization of These Notes

The notes start with the definition and computation of basic “static” thermodynamic quantities, such as the partition function and free energy density, in various settings. Considered are in turn quantum mechanics (Sect. 1), free and interacting scalar field theories (Sects. 2 and 3, respectively), fermionic systems (Sect. 4), and gauge fields (Sect. 5). The main points of these sections include the introduction of the so-called imaginary-time formalism, the functioning of renormalization at finite temperature, and the issue of infrared problems that complicates almost every computation in relativistic thermal field theory. The last of these issues leads us to introduce the concept of effective field theories (Sect. 6), after which we consider the changes caused by the introduction of a finite density or chemical potential (Sect. 7). After these topics, we move on to a new set of observables, so-called real-time quantities, which play an essential role in many modern phenomenological applications of thermal field theory (Sect. 8). In the final chapter of the book, a number of concrete applications of the techniques introduced are discussed in some detail (Sect. 9).

We note that Sects. 1–7 are presented on an elementary and self-contained level and require no background knowledge beyond statistical physics, quantum mechanics, and rudiments of quantum field theory. They could constitute the contents of a one-semester basic introduction to perturbative thermal field theory. In Sect. 8, the level increases gradually, and parts of the discussion in Sect. 9 are already close to the research level, requiring more background knowledge. Conceivably the topics of Sects. 8 and 9 could be covered in an advanced course on perturbative thermal field theory or in a graduate student seminar. In addition the whole book is suitable for self-study and is then advised to be read in the order in which the material has been presented.

Recommended Literature

A pedagogical presentation of thermal field theory, concentrating mostly on Euclidean observables and the imaginary-time formalism, can be found in Ref. [1]. The current notes borrow significantly from this classic treatise.

In thermal field theory, the community is somewhat divided between those who find the imaginary-time formalism more practicable and those who prefer to use the so-called real-time formalism from the beginning. Particularly for the latter community, the standard reference is Ref. [2], which also contains an introduction to particle production rate computations.

A modern textbook, partly an update of Ref. [1] but including also a full account of real-time observables, as well as reviews on many recent developments, is provided by Ref. [3].

Lucid lecture notes on transport coefficients, infrared resummations, and nonequilibrium phenomena such as thermalization can be found in Ref. [4]. Worthy reviews with varying foci are offered by Refs. [5–11].

Finally, an extensive review of current efforts to approach a non-perturbative understanding of real-time thermal field theory has been presented in Ref. [12].

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Contents