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Robin Stanley Johnson • Gabriele Villari

Nonlinear Water Waves

Cetraro, Italy 2013

Adrian Constantin
Editor

Springer

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Preface

The study of water waves involves various disciplines such as mathematics, physics and engineering—to name the obvious—and within this, there are many specific areas of direct or associated interest such as pure mathematics, applied mathematics, modelling, numerical simulation, laboratory experiments, data collection in the field, the design and construction of ships, harbours and offshore platforms, the prediction of natural disasters (e.g. tsunamis), climate studies and so on. We are all familiar with, and probably excited by, the experience of seeing waves in lakes, rivers, oceans and even baths and sinks; they are often beautiful, but sometimes terrifying. They are also mathematically intriguing and susceptible to a number of different, but very particular, theoretical approaches. All these various studies help us to improve, in one way or another, our understanding of wave propagation which, in turn, inevitably leads to better physics and engineering as we work with, and deal with the effects of, waves on water.

The meeting held in Cetraro, Italy, June 24–28, 2013, under the auspice of, and supported by, the Centro Internazionale Matematico Estivo (C.I.M.E., the International Mathematical Summer Centre) aimed to present some of the current mathematical research in this area. The summer school provided a vehicle for a selection of the main mathematical avenues to be presented via a series of lectures; in addition, there were short presentations and much discussion, covering other related topics, such as numerical methods and modelling. This volume brings together the four main lecture courses. The intention, through the lectures, was to present quite a range of mathematical ideas, primarily to show what is possible and what, currently, is of particular interest. The general background to the mathematical formulation of the classical water-wave problem, and the interplay between what is observed and how we model this using a robust mathematical formulation, appears in 'Asymptotic methods for weakly nonlinear and other water waves' by R.S. Johnson. These lectures also covered some aspects of the construction and generalisation of soliton-type equations (including a brief introduction to 'soliton theory') and, of particular current interest, the rôle that background vorticity can play in the evolution of the waves and its effects upon their properties. In order to show the wealth of possibilities using an asymptotic approach, periodic waves with vorticity and edge waves are also discussed. The lectures given by A. Constantin, entitled ‘Exact travelling periodic water waves in two-dimensional irrotational flows’, also explain the connection between what is observed (both in the laboratory and in the field) and what we can describe and predict using a mathematical approach. The exact solutions are described by, for example, the properties of the associated particle paths based on harmonic analysis and the theory of elliptic partial differential equations. Thus the very practical essence of the waves and very powerful and rigorous techniques are moulded to produce a comprehensive picture of the types of flows associated with classical water waves. The theme of particle paths is taken up in ‘A survival kit in phase plane analysis: some basic models and problems’ by G. Villari, but the approach here is to use another fundamental mathematical tool: the method of phase-plane analysis. Again, the thrust is to show how a sophisticated and familiar branch of mathematics can relate to, and usefully describe, the details of the complex flow patterns that are observed. The final series of lectures made use of the very powerful ideas that underpin the modern techniques of functional analysis. J. Escher discussed the nature of wave breaking as it applies, mainly, to the solutions of the Camassa-Holm equation in ‘Breaking water waves’. One of the exciting properties of this model equation is that it captures both the non-breaking and breaking wave phenomena of classical water waves. Some details that help to explain the rôle of the initial data in predicting the final development of the wave are provided, producing some important estimates.

These four lectures provide a useful source for those who want to begin to investigate how mathematics can be used to improve our understanding of this rapidly developing classical research area. In addition, some of the material can be used by those who are already familiar with one branch of the study of water waves, to learn more about other areas. We therefore commend this collection of lectures to both the novice and the expert.

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