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Quantum Field Theory I

Foundations and Abelian and Non-Abelian Gauge Theories

Springer

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Preface to Volume I

This textbook is based on lectures given in quantum field theory (QFT) over the years to graduate students in theoretical and experimental physics. The writing of the book spread over three continents: North America (Canada), Europe (Ireland), and Asia (Thailand). QFT was born about 90 years ago, when quantum mechanics met relativity, and is still going strong. The book covers, pedagogically, the wide spectrum of developments in QFT emphasizing, however, those parts which are reasonably well understood and for which satisfactory theoretical descriptions have been given.

The legendary Richard Feynman *in* his 1958 Cornell, 1959–1960 Cal Tech lectures on QFT of fundamental processes, the first statement he makes, the very first one, is that the *lectures cover all of physics*.[1] One quickly understands what Feynman meant by covering all of physics. The role of fundamental physics is to describe the basic interactions of Nature and *QFT, par excellence, is supposed to do just that*. Feynman's statement is obviously more relevant today than it was then, since the recent common goal is to provide a unified description of *all* the fundamental interactions in nature.

The book requires as background a good knowledge of quantum mechanics, including rudiments of the Dirac equation, as well as elements of the Klein-Gordon equation, and the reader would benefit much by reading relevant sections of my earlier book : *Quantum Theory: A Wide Spectrum (2006), Springer* in this respect.

This book differs from QFT books that have appeared in recent years[2] in several respects and, in particular, it offers something new in its approach to the subject, and the reader has plenty of opportunity to be exposed to many topics not covered, or

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[1] R. P. Feynman, The Theory of Fundamental Processes, The Benjamin/Cummings Publishing Co., Menlo Park, California. 6th Printing (1982), page 1.

[2] Some of the fine books that I am familiar with are: L. H. Ryder, Quantum Field Theory; S. Weinberg, The Quantum Theory of Fields I (1995) & II (1996), Cambridge: Cambridge University Press; M. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory, New York: Westview Press (1995); B. DeWitt, The Global Approach to Quantum Field Theory, Oxford: Oxford University Press (2014).

just touched upon, in standard references. Some notable differences are seen, partly, from unique features in the following material included in ours:

- The very elegant functional *differential* approach of Schwinger, referred to as the quantum dynamical (action) principle, and its underlying theory are used systematically in generating the so-called vacuum-to-vacuum transition amplitude of both abelian and non-abelian gauge theories, in addition to the well-known functional integral approach of Feynman, referred to as the path-integral approach, which are simply related by functional Fourier transforms and delta functionals.
- Transition amplitudes are readily extracted by a direct expansion of the vacuum-to-vacuum transition amplitude in terms of a unitarity sum, which is most closely related to actual experimental setups with particles emitted and detected prior and after a given process and thus represent the underlying physics in the clearest possible way.
- Particular emphasis is put on the concept of a quantum field and its particle content, both physically and technically, as providing an appropriate description of physical processes at sufficiently high energies, for which relativity becomes the indispensable language to do physics and explains the exchange that takes place between energy and matter, allowing the creation of an unlimited number of particles such that the number of particles need not be conserved, and for which a variable number of particles may be created or destroyed. Moreover, quantum mechanics implies that a wavefunction renormalization arises in QFT field independent of any perturbation theory – a point not sufficiently emphasized in the literature.
- The rationale of the stationary action principle and emergence of field equations, via field variations of transformation functions and generators of field variations. The introduction of such generators lead, self consistently, to the field equations. Such questions are addressed as: “Why is the variation of the action, *within* the boundaries of transformation functions, set equal to *zero* which eventually leads to the Euler-Lagrange equations?”, “How does the Lagrangian density appear in the formalism?” “What is the significance in commuting/anti-commuting field components within the interaction Lagrangian density in a theory involving field operators?” These are some of the questions many students seem to worry about.
- A panorama of all the fields encountered in present high-energy physics, together with the details of the underlying derivations are given.
- Schwinger’s point splitting method of currents is developed systematically in studying abelian and non-abelian gauge theories anomalies. Moreover, an explicit experimental test of the presence of an anomaly is shown by an example.
- Derivation of the Spin & Statistics connection and CPT symmetry, emphasizing for the latter that the invariance of the action under CPT transformation is not sufficient for CPT symmetry, but one has also to consider the roles of incoming and outgoing particles.

- The fine-structure effective coupling $\alpha \simeq 1/128$ at high energy corresponding to the mass of the neutral Z^0 vector boson based on all the charged leptons and all those contributing quarks of the three generations.
- Emphasis is put on renormalization theory, including its underlying general subtractions scheme, often neglected in treatments of QFT.
- Elementary derivation of Faddeev-Popov factors directly from the functional differential formalism, with constraints, and their *modifications*, and how they may even arise in some abelian gauge theories.
- A fairly detailed presentation is given of "deep inelastic" experiments as a fundamental application of quantum chromodynamics.
- Schwinger line integrals, origin of Wilson loops, lattices, and quark confinement.
- Neutrino oscillations,[3] neutrino masses, neutrino mass differences, and the "seesaw mechanism."
- QCD jets and parton splitting, including gluon splitting to gluons.
- Equal importance is put on both abelian and non-abelian gauge theories, witnessing the wealth of information also stored in the abelian case.[4]
- A most important, fairly detailed, and semi-technical introductory chapter is given which traces the development of QFT since its birth in 1926 without tears, in abelian and non-abelian gauge theories, including aspects of quantum gravity, as well as examining the impact of supersymmetry, string theory, and the development of the theory of renormalization, as a *pedagogical* strategy for the reader to be able to master the basic ideas of the subject at the outset before they are encountered in glorious technical details later.
- *Solutions* to all the problems are given right at the end of the book.

With the mathematical rigor that renormalization has met over the years and the reasonable agreement between gauge theories and experiments, the underlying theories are in pretty good shape. This volume is organized as follows. The first introductory chapter traces the subject of QFT since its birth, elaborating on many of its important developments which are conveniently described in a fairly simple language and will be quite useful for understanding the underlying technical details of the theory covered in later chapters including those in Volume II. A preliminary chapter follows which includes the study of symmetry transformations in the quantum world, as well as of intricacies of functional differentiation and functional integration which are of great importance in field theory. Chapter 3 deals with quantum field theory methods of spin 1/2 culminating in the study of anomalies in the quantum world. The latter refers to the fact that a conservation law

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[3] It is rather interesting to point out that the theory of neutrino oscillations was written up in this book much earlier than the 2015 Nobel Prize in Physics was announced on neutrino oscillations.

[4] With the development of non-abelian gauge theories, unfortunately, it seems that some students are not even exposed to such derivations as of the "Lamb shift" and of the "anomalous magnetic moment of the electron" in QED.

in classical physics does not necessarily hold in the quantum world. Chapter 4, a critical one, deals with the concept of a quantum field, the Poincaré algebra, and particle states. Particular attention is given to the stationary action principle as well as in developing the solutions of QFT via the quantum dynamical principle. This chapter includes the two celebrated theorems dealing with CPT symmetry and of the Spin & Statistics connection. A detailed section is involved with the basic quantum fields one encounters in present day high-energy/elementary-particle physics and should provide a useful reference source for the reader. Chapter 5 treats abelian gauge theories (QED, scalar boson electrodynamics) in quite details and includes, in particular, the derivations of two of the celebrated results of QED which are the anomalous magnetic moment of the electron and the Lamb shift. Chapter 6 is involved with non-abelian gauge theories (electroweak, QCD, Grand unification).[5] Such important topics are included as "asymptotic freedom," "deep inelastic" scattering, QCD jets, parton splittings, neutrino oscillations, the "seesaw mechanism" and neutrino masses, Schwinger-line integrals, Wilson loops, lattices, and quark confinement. Unification of coupling parameters of the electroweak theory and of QCD are also studied, as well as of spontaneous symmetry breaking in both abelian and non-abelian gauge theories, and of renormalizability aspects of both gauge theories, emphasizing the so-called BRS transformations for the latter. We make it a point, pedagogically, to derive things in detail, and some of such details are *relegated* to appendices at the end of the respective chapters with the main results *given* in the sections in question. Five general appendices, at the end of this volume, cover some additional important topics and/or technical details. In particular, I have included an appendix covering some aspects of the general theory of renormalization and its underlying subtractions scheme itself which is often neglected in books on QFT. Fortunately, my earlier book, with *proofs* not just words, devoted completely to renormalization theory – *Renormalization (1983), Academic Press* – may be consulted for more details. The problems given at the end of the chapters form an integral part of the book, and many developments in the text depend on the problems and may include, in turn, additional material. They should be attempted by every serious student. *Solutions* to all the problems are given right at the end of the book for the convenience of the reader. The introductory chapter together with the introductions to each chapter provide the motivation and the *pedagogical* means to handle the technicalities that follow them in the texts.

I hope this book will be useful for a wide range of readers. In particular, I hope that physics graduate students, not only in quantum field theory and high-energy physics, but also in other areas of specializations will also benefit from it as, according to my experience, they seem to have been left out of this fundamental area of physics, as well as instructors and researchers in theoretical physics. The content of this volume may be covered in one-year (two semesters) quantum field theory courses.

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[5] QED and QCD stand, respectively, for quantum electrodynamics and quantum chromodynamics.

In Volume II, the reader is introduced to quantum gravity, supersymmetry, and string theory,[6] which although may, to some extent, be independently read by a reader with a good background in field theory, the present volume sets up the language, the notation, provides additional background for introducing these topics, and will certainly make it much easier for the reader to follow. In this two-volume set, aiming for completeness in covering the basics of the subject, I have included topics from the so-called conventional field theory (the classics) to ones from the modern or the new physics which I believe that every serious graduate student studying quantum field theory should be exposed to.

Without further ado, and with all due respect to the legendary song writer Cole Porter, let us find out "what is this thing called QFT?"

Edouard B. Manoukian

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[6] Entitled: Quantum Field Theory II: *Introductions to Quantum Gravity, Supersymmetry, and String Theory"* (2016), Springer.

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In the beginning of it all, I was introduced to the theoretical aspects of quantum field theory by Theodore Morris and Harry C. S. Lam, both from McGill and to its mathematical intricacies by Eduard Prugovečki from the University of Toronto. I am eternally grateful to them. Over the years, I was fortunate enough to attend a few lectures by Julian Schwinger and benefited much from his writings as well. Attending a lecture by Schwinger was quite an event. His unique elegant, incisive, physically clear approach and, to top it off, short derivations were impressive. When I was a graduate student, I would constantly hear that Schwinger "does no mistakes." It took me years and years to understand what that meant. My understanding of this is because he had developed such a powerful formalism to do field theory that, unlike some other approaches, everything in the theory came out automatically and readily without the need to worry about multiplicative factors in computations, such as 2π's and other numerical factors, and, on top of this, is relatively easy to apply. Needless to say this has much influenced my own approach to the subject. He had one of the greatest minds in theoretical physics of our time.

I want to take this opportunity as well to thank Steven Weinberg, the late Abdus Salam, Raymond Streater, and Eberhard Zeidler for the keen interest they have shown in my work on renormalization theory.

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Contents

Notation and Data

- Latin indices $i, j, k, \ldots$ are generally taken to run over 1,2,3, while the Greek indices $\mu, \nu, \ldots$ over $0, 1, 2, 3$ in 4D. Variations do occur when there are many different types of indices to be used, and the meanings should be evident from the presentations.
- The Minkowski metric $\eta_{\mu\nu}$ is defined by $[\eta_{\mu\nu}] = \mathrm{diag}[-1, 1, 1, 1] = [\eta^{\mu\nu}]$ in 4D.
- Unless otherwise stated, the fundamental constants $\hbar$, c are set equal to one.
- The gamma matrices satisfy the anti-commutation relations $\{\gamma^{\mu}, \gamma^{\nu}\} = -2\,\eta^{\mu\nu}$.
- The Dirac, the Majorana, and the chiral representations of the γ^{μ} matrices are defined in Appendix I at the end of the book.
- The charge conjugation matrix is defined by $\mathscr{C} = \mathrm{i}\gamma^{2}\gamma^{0}$.
- $\overline{\psi} = \psi^{\dagger}\gamma^{0}$, $\overline{u} = u^{\dagger}\gamma^{0}$, $\overline{v} = v^{\dagger}\gamma^{0}$. A Hermitian conjugate of a matrix M is denoted by $M^{\dagger}$, while its complex conjugate is denoted by M^{*}.
- The step function is denoted by $\theta(x)$ which is equal to 1 for $x > 0$, and 0 for $x < 0$.
- The symbol ε is used in dimensional regularization (see Appendix III). ϵ is used in defining the boundary condition in the denominator of a propagator $(Q^2 + m^2 - \mathrm{i}\,\epsilon)$ and should not be confused with ε used in dimensional regularization. We may also use either one when dealing with an infinitesimal quantity, in general, with ϵ more frequently, and this should be self-evident from the underlying context.
- For units and experimental data, see the compilation of the "Particle Data Group": Beringer et al. [1] and Olive et al. [2]. The following (some obviously approximate) numerical values should, however, be noted:

$$\begin{aligned} 1\ \mathrm{MeV} &= 10^{6}\ \mathrm{eV} \\ 1\ \mathrm{GeV} &= 10^{3}\ \mathrm{MeV} \\ 10^{3}\ \mathrm{GeV} &= 1\ \mathrm{TeV} \\ 1\ \mathrm{erg} &= 10^{-7}\ \mathrm{J} \end{aligned}$$

$$\begin{aligned}1\ \mathrm{J} &= 6.242\times 10^{9}\ \mathrm{GeV}\\ \mathrm{c} &= 2.99792458\times 10^{10}\ \mathrm{cm/s\ (exact)}\\ \hbar &= 1.055\times 10^{-34}\ \mathrm{J\ s}\\ \hbar\,\mathrm{c} &= 197.33\ \mathrm{MeV\ fm}\\ 1\ \mathrm{fm} &= 10^{-13}\ \mathrm{cm}\end{aligned}$$

(Masses) $M_p = 938.3\,\mathrm{MeV/c^2}$, $M_n = 939.6\,\mathrm{MeV/c^2}$,
$M_W = 80.4\ \mathrm{GeV/c^2}$, $M_Z = 91.2\,\mathrm{GeV/c^2}$,
$m_e = 0.511\,\mathrm{MeV/c^2}$, $m_\mu = 105.66\ \mathrm{MeV/c^2}$, $m_\tau = 1777\,\mathrm{MeV/c^2}$.
Mass of $\nu_e < 2\,\mathrm{eV/c^2}$, Mass of $\nu_\mu < 0.19\,\mathrm{MeV/c^2}$, Mass of $\nu_\tau < 18.2\,\mathrm{MeV/c^2}$,
Mass of the neutral Higgs $H^0 \approx 125.5\,\mathrm{GeV/c^2}$.
For approximate mass values of some of the quarks taken, see Table 5.1 in Sect. 5.19.2. For more precise range of values, see Olive et al. [2].
(Newton's gravitational constant) $\mathrm{G_N} = 6.709\times 10^{-39}\,\hbar\,\mathrm{c^5/\,GeV^2}$.
(Fermi weak interaction constant) $\mathrm{G_F} = 1.666\times 10^{-5}\,\hbar^3\,\mathrm{c^3/\,GeV^2}$.
Planck mass $\sqrt{\hbar\,\mathrm{c/G_N}} \approx 1.221\times 10^{19}\,\mathrm{GeV/c^2}$,
Planck length $\sqrt{\hbar\,\mathrm{G_N/c^3}} \approx 1.616\times 10^{-33}\ \mathrm{cm}$.
Fine structure constant $\alpha = 1/137.04$ at $Q^2 = 0$, and $\approx 1/128$ at $Q^2 \approx M_Z^2$.
For the weak-mixing angle θ_W, $\sin^2\theta_W \approx 0.232$, at $Q^2 \approx M_Z^2$.
$\alpha/\sin^2\theta_W \approx 0.034$, at $Q^2 \approx M_Z^2$.
Strong coupling constant $\alpha_s \approx 0.119$, at $Q^2 \approx M_Z^2$.

References

1. Beringer, J., et al. (2012). Particle data group. *Physical Review D, 86*, 010001.
2. Olive, K. A., et al. (2014). Particle data group. *Chinese Physics C, 38*, 090001.