

# Quantifiers and Cognition: Logical and Computational Perspectives

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# Quantifiers and Cognition: Logical and Computational Perspectives

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The book draws heavily on material from my dissertation that at places overlaps with related publications. In particular, let me mention the following: Chapter 8 of the book reproduces in part material from Chapter 4 of my dissertation that was also published in my 2010 article *Linguistics & Philosophy*. Chapter 9 of the book has its source in Chapter 6 of the dissertation that was a base for Szymanik and Gierasimczuk's 2009 paper in *Journal of Semantics*. Chapter 10 of the book reproduces and expands on Chapter 5 of my dissertation and 2008 *Journal of Logic, Language, and Information* paper by Kontinen and Szymanik. It also reports our joint technical work, that answers some of the open questions from my dissertation, that was published in 2013 paper in *Lecture Notes in Artificial Intelligence*, 6642.

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# Introduction

There are at least two levels of language complexity—the syntactic one and the semantic one. This book is about semantics, with a focus on the *complexity of meaning of natural language quantifiers*. The general question we aim to answer is why the meanings of some sentences are more difficult than the meanings of others, for instance, why we probably all agree that it is easier to evaluate sentence (1) than to evaluate sentence (2) and why sentence (3) seems hard while sentence (4) sounds odd.

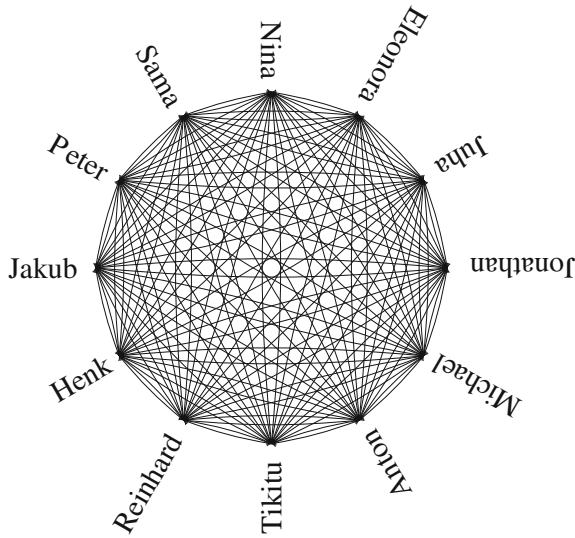
- (1) Every book on the shelf is yellow.
- (2) Most of the books on the shelf are yellow.
- (3) Less than half of the members of parliament refer to each other.
- (4) Some book by every author is referred to in some essay by every critic.

The tools of logic and computability theory are useful in making such differences precise. One can formalize linguistic expressions in logic, study their mathematical properties, and ask about the empirical value of formal models. To this end the book provides some *mathematical results on the complexity of semantics*, *linguistic case studies*, and *psycholinguistic empirical experiments*. To make these notions workable, this study focuses on quantifier sentences such as (1)–(4).

The complexity analysis of the quantifier sentences in natural language allow us to make and test empirical predictions about the *cognitive difficulty* of language processing, and about specific *cognitive resources* involved in it. The logical and cognitive parts of the book are tightly interwoven with more methodological and philosophical discussions on how to relate formal claims on complexity to linguistic and cognitive realities. We confront and combine theoretical work with experiments. Such an approach allows us to characterize semantic concepts captured by natural language and study the limits of its expressivity: what makes some semantic concepts fall outside everyday language?

This book is *highly interdisciplinary*. It contains studies in logic, natural language semantics, philosophy of mind and language, theoretical computer science, cognitive science, and their intersections.

\*\*\*



**Fig. 1** A model of the sentence ‘Everyone knows everyone else’. The names are connected by a link if two people know each other

The *theory of quantifiers* is fascinating from a complexity point of view. There is an intimate relationship between the complexity of quantifier sentences and the expressive power of language. For instance, imagine that you want to say that there is the following pattern of acquaintance (illustrated in the Fig. 1) among some group of your friends:

At first glance it seems like a lot needs to be said. However, with the use of quantifiers the situation is very easy to describe. It is just enough to say that within the group:

(5) Everyone knows everyone else.

Imagine, however, that if we were speaking a language without quantification, then suddenly the task would become much harder, as there would be no succinct way of expressing the complete friendship relation, and one would need to use a very long conjunction:

(6) Jakob knows Peter and Jakob knows Sama and ... and Peter knows Sama and ... and Henk knows Jakob.

Quantifiers relieve us of such linguistic burdens.