

## Part II. Vertical connections and the twisted prolongations.

In this, geometrical, part of this work, we relate the procedures of  $K$ -twisted prolongations of vector fields to the "vertical connections" at the 1-jet bundle  $J^1(\pi) \rightarrow Y$  over the the configurational space  $Y$ . Later on, we extend  $K$ -twisted prolongation and the geometrical interpretation of such prolongations to the  $k$ -jet bundles  $J^k(\pi)$  with  $1 \leq k \leq \infty$ .

In all cases it is (Ehresmann) connections in the bundle  $\pi_{10} : J^1(\pi) \rightarrow Y$  (respectively, in the  $k$ -jet bundle  $J^k(\pi) \rightarrow Y$ ) (see Appendix II) that plays the main role. This opens a way to recognize and study existing relations of the procedures of non-commutative prolongations of variations and the related properties of the geometrical structures on the jet bundles  $J^k(\pi)$  (e.g., connections, curvature and torsion of connections, contact structures on the jet bundles, etc.).

To follow the presentation of this theme, a knowledge of basic geometrical notions and constructions of the Variational Calculus is necessary. That is why in the Appendix I of this text we provide a short description of basic geometrical structures necessary in Chapters 3,4 - fiber bundles, Ehresmann connections, curvature, linear and affine connections, their curvature and torsion, automorphisms of bundles, absolute parallelism. In the second part of Appendix - Appendix II we present basic information about of the jet bundles including their contact structures, Lie fields, flow prolongations of vector fields and Ehresmann connections in the jet bundles. Below we will be using introduced geometrical notions and results refereing to the Appendices I,II and the references therein.