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Heat Transfer of Laminar Mixed Convection of Liquid

Springer

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Preface

Three years ago, just after our book “Free convection film flows and heat transfer—models of laminar free convection with phase change for heat and mass transfer analysis” was published, our research interests were transferred to the more challenging field of laminar mixed convection heat and mass transfer. This is a cross field of free and forced convection with more difficult issues than free or forced convections. Strictly speaking, all forced convection heat transfer is mixed convection heat transfer. In fact, net forced convection heat transfer does never exist. It is because in fluid medium of so-called forced convection heat transfer, there should be temperature difference, which leads to fluid buoyancy. While, the fluid buoyancy will cause free convection. There have been numerous previous studies on mixed convection, however, for such a difficult field, engineers still have to deal with many unresolved problems. Due to these unresolved difficult problems, so far there is a lack of studies on the development of reliable theoretical or experiment achievements for heat transfer application of mixed convection. These difficulties prompted us to think deeply. Finally, we knew that to resolve these problems we needed to focus on the following issues: (i) developing an advanced theoretical and mathematical model; (ii) carefully considering and treating the variable physical properties of fluids; (iii) obtaining systems of rigorous numerical solutions; and (iv) on these bases, creating optimal formalized equations of heat transfer coefficient. In fact, these focused issues are interrelated. An advanced theoretical and mathematical model is the basic condition for successful investigation of mixed convection heat transfer. To address this issue, our innovative similarity transformation on velocity field would be taken as a better alternative. While, in our system of studies on free and forced convection heat and mass transfer, our innovative similarity transformation was conveniently and reliably applied. On the other hand, the variable physical properties will be taken into account in this study, and the polynomial model will be induced for treatment of the variable physical properties of liquids. Although there could be different models selected for consideration of the variable physical properties of liquids, the polynomial model should be the most convenient one combined with the transformed governing ordinary differential equations used for simultaneous solutions.Through our comparison on the values of mixed convection heat transfer coefficient predicted for consideration of coupled effect of variable physical properties with those for consideration of Boussinesq approximation, we found their deviations are as high as over 8.4 %. It proves that proper consideration of the variable physical properties of fluids is very important for enhancement of the theoretical and practical value of convection heat transfer. Hence, it is important to face up the variable physical properties of fluids in rigorous theoretical study on clarification of convection heat transfer. Actually, since two decades ago, we have been committed to consider effect of fluid's variable physical properties on convection heat transfer in our studies. We deeply feel that it is necessary to consider coupled effect of variable physical properties for assurance of the theoretical and practical value for research of convection heat transfer. While, Boussinesq approximation or ignoring fluid's variable physical properties will severely weakened the theoretical and practical value of convection heat transfer. Furthermore, creation of the optimal formalized equations of heat transfer coefficient will be an important indicator on study with theoretical and application value for convection heat transfer application.

With this opportunity we thank Springer-Verlag for publishing our book "Heat Transfer of Laminar Mixed Convection of Liquid". We hope, based on our above efforts, this book will have a special theoretical and practical value for mixed convection heat transfer application, and will be of benefit to engineers, researchers, professors, and Ph.D. and master students in their engineering design, research work and teaching courses on thermal fluids and engineering fields related to convection heat and mass transfer.

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Appendix A: Tables with Physical Properties 215

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Nomenclature

A	Area, m^2
c_p	Specific heat, J/(kg K)
g	Gravity acceleration, m/s^2
$Gr_{x,f}$	Local Grashof number for consideration of Boussinesq approximation
$Gr_{x,\infty}$	Local Grashof number for consideration of variable physical properties
$Mc_{x,f}$	Local mixed convection parameter for consideration of Boussinesq approximation
$Mc_{x,\infty}$	Local mixed convection parameter for consideration of variable physical properties
$Nu_{x,f}$	Local Nusselt number for consideration of Boussinesq approximation
$\overline{Nu}_{x,f}$	Average Nussel number for consideration of Boussinesq approximation
$Nu_{x,w}$	Local Nusselt number for consideration of variable physical properties
$\overline{Nu}_{x,w}$	Average Nusselt number for consideration of variable physical properties
Pr	Prandtl number
Pr_f	Mean Prandtl number at average temperature
Pr_w	Local Prandtl number at wall temperature
Pr_∞	Local Prandtl number at bulk local temperature
Q	Heat transfer rate, W
$q_{x,f}$	Local heat transfer rate for consideration of Boussinesq approximation, W/m^2
$Q_{x,f}$	Total heat transfer rate for consideration of Boussinesq approximation, W
$q_{x,w}$	Local heat transfer rate for consideration of variable physical properties, W/m^2

$Q_{x,w}$	Tocal heat transfer rate for consideration of variable physical properties, W
$Re_{x,f}$	Local Reynolds number with consideration of Boussinesq approximation
$Re_{x,\infty}$	Local Reynolds number with consideration of variable physical properties
t	Temperature, ^{o}C
T	Absolute temperature, K
Δt	Temperature difference, $t_w - t_\infty$, ^{o}C
t_f	Average temperature, ^{o}C
t_w	Wall temperature, ^{o}C
t_∞	Fluid bulk temperature, ^{o}C
w_x, w_y	Velocity components respectively in x and y directions, m/s
$w_{x,\infty}$	Velocity component beyond the boundary layer, m/s
$W_x(\eta, x)$, $W_y(\eta, x)$	Similarity velocity component, m/s
$W_x(\eta, Mc_{x,f}), /, W_y(\eta, Mc_{x,f})$	Equivalent similarity velocity components with consideration of Boussinesq approximation, m/s
$W_x(\eta, Mc_{x,\infty}), /, W_y(\eta, Mc_{x,\infty})$	Equivalent similarity velocity components with consideration of variable physical properties
x, y	Two-dimensional coordinate variables

Greek Symbols

ρ	Density, kg/m^3
μ	Absolute viscosity, kg/(m s)
λ	Thermal conductivity, W/(m K)
v	Kinetic viscosity, m^2/s
β	Expansion coefficient, 1/K
η	Similarity coordinate variable
θ	Similarity temperature
$\alpha_{x,f}$	Local heat transfer coefficient with consideration of Boussinesq approximation, W/(m^2 K)
$\overline{\alpha}_{x,f}$	Average heat transfer coefficient with consideration of Boussinesq approximation, W/(m^2 K)
$\alpha_{x,w}$	Local heat transfer coefficient with consideration of variable physical properties, W/(m^2 K)
$\overline{\alpha}_{x,w}$	Average heat transfer coefficient with consideration of variable physical properties, W/(m^2 K)
$\left(-\frac{\partial\theta(\eta,Mc)}{\partial\eta}\right)_{\eta=0}$	Wall similarity temperature gradient for consideration of Boussinesq approximation

$\left(-\frac{\partial\theta(\eta,Mc)}{\partial\eta}\right)_{\eta=0}$	Wall similarity temperature gradient for consideration of variable physical properties
$\frac{1}{\rho}\frac{d\rho}{d\eta}$	Fluid density factor
$\frac{1}{\mu}\frac{d\mu}{d\eta}$	Fluid viscosity factor
$\frac{1}{\lambda}\frac{d\lambda}{d\eta}$	Fluid thermal conductivity factor

Subscripts

f	At average temperature
w	At wall
∞	In fluid bulk