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Mathematics for Natural Scientists II

Advanced Methods

Springer

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Preface

This is the second volume of the course of mathematics for natural scientists. It is loosely based on the mathematics course for second year physics students at King's College London that I have been reading for more than 10 years. It follows the spirit of the first volume [1] by continuing a gradual build-up of the mathematical knowledge necessary for, but not exclusively, physics students.

This volume covers more advanced material, beginning with two essential components: linear algebra (Chap. 1) and theory of functions of complex variables (Chap. 2). These techniques are heavily used in the chapters that follow. Fourier series are considered in Chap. 3, special functions of mathematical physics (Dirac delta function, gamma and beta functions, detailed treatment of orthogonal polynomials, the hypergeometric differential equation, spherical and Bessel functions) in Chap. 4 and then Fourier (Chap. 5) and Laplace (Chap. 6) transforms. In Chap. 7, a detailed treatment of curvilinear coordinates is given, including the corresponding differential calculus. This is essential, as many physical problems possess symmetry and using appropriate curvilinear coordinates may significantly simplify the solution of the corresponding partial differential equations (studied in Chap. 8) if the symmetry of the problem at hand is taken into account. The book is concluded with variational calculus in Chap. 9.

As in the first volume, I have tried to introduce new concepts gradually and as clearly as possible, giving examples and problems to illustrate the material. Across the text, all the proofs necessary to understand and appreciate the mathematics involved are also given. In most cases, the proofs would satisfy the most demanding physicist or even a mathematician; only in a few cases have I had to sacrifice the "strict mathematical rigour" by presenting somewhat simplified derivations and/or proofs.

As in the first volume, many problems are given throughout the text. These are designed mainly to illustrate the theoretical material and require the reader to complete them in order to be in a position to move forward. In addition, other problems are offered for practise, although I have to accept, their number could have been larger. For more problems, the reader is advised to consult other texts, e.g. the books [2–6].

When working on this volume, I have mostly consulted a number of excellent classic Russian textbooks [7–12]. As far as I am aware, some of them are available in English, and I would advise a diligent student to continue his/her education by reading these. Concerning the others, there is of course an obvious language barrier. Unfortunately, as I cannot ask the reader to learn Russian purely for that purpose, these texts remain inaccessible for most readers. I hope that the reader would be able to find more specialised texts in English, which go beyond the scope of this (and the previous) book to further the development of their studies, e.g. books [13–26] represent a rather good selection which cover the topics of this volume, but this list of course by no means is complete.

The mathematics throughout the book is heavily illustrated by examples from condensed matter physics. In fact, probably over a quarter of the text of the whole volume is occupied by these. Every chapter, save Chap. 8, contains a large concluding section exploring physics topics that necessitate the mathematics presented in that chapter. Chapter 8, on partial differential equations, is somewhat special in this respect, as it is entirely devoted to solving equations of mathematical physics (wave, Laplace and heat transport equations). Consequently, it does not have a special section on applications. When selecting the examples from physics, I was mostly governed by my own experience and research interests as well as several texts, such as the books [27, 28]. In fact, examples from [27] have been used in both of these volumes.

As in the first volume, this book begins with a list of the names of all the scientists across the world, mathematicians, physicists and engineers, whose invaluable contribution formed the foundation of the beautiful sciences of mathematics and physics that have been enjoying a special bond throughout the centuries.

Should you find any errors or unnoticed misprints, please send your corrections either directly to myself (lev.kantorovitch@kcl.ac.uk) or to the publisher. Your general comments, suggestions and any criticism related to these two volumes would be greatly appreciated.

This book concludes the project I started about four years ago, mainly working in the evenings, at the weekends, as well as on the train going to and from work. Not everything I initially planned to include has appeared in the books, although the two volumes contain most of the essential ideas young physicists, engineers and computational chemists should become familiar with. Theory of operators, group theory, tensor calculus, stochastic theory (or theory of probabilities) and some other more specialised topics have not been included, but can be found in multiple other texts. Still, I hope these volumes will serve as an enjoyable introduction to the beautiful world of mathematics for students, encouraging them to think more and ask for more. I am also confident that the books will serve as a rich source for lecturers.

I wish you happy reading!

London, UK Lev Kantorovich

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Famous Scientists Mentioned in the Book

Throughout the book various people, both mathematicians and physicists, who are remembered for their outstanding contribution in developing science, will be mentioned. For reader's convenience, their names (together with some information borrowed from their Wikipedia pages) are listed here in the order they first appear in the text:

Leopold Kronecker (1823–1891) was a German mathematician.

Georg Friedrich Bernhard Riemann (1826–1866) was an influential German mathematician who made lasting and revolutionary contributions to analysis, number theory and differential geometry.

Jørgen Pedersen Gram (1850–1916) was a Danish actuary and mathematician.

Erhard Schmidt (1876–1959) was an Estonian-German mathematician.

Adrien-Marie Legendre (1752–1833) was a French mathematician.

Edmond Nicolas Laguerre (1834–1886) was a French mathematician.

Charles Hermite (1822–1901) was a French mathematician.

Pafnuty Lvovich Chebyshev (1821–1894) was a Russian mathematician.

Albert Einstein (1879–1955) was a German-born theoretical physicist who developed the general theory of relativity and also contributed in many other areas of physics. He received the 1921 Nobel Prize in physics for his "services to theoretical physics".

Wolfgang Ernst Pauli (1900–1958) was an Austrian theoretical physicist and one of the pioneers of quantum physics.

Jean-Baptiste Joseph Fourier (1768–1830) was a French mathematician and physicist.

Gabriel Cramer (1704–1752) was a Swiss mathematician.

Hendrik Antoon Lorentz (1853–1928) was a Dutch physicist.

Józef Maria Hoene-Wroński (1776–1853) was a Polish Messianist philosopher, mathematician, physicist, inventor, lawyer and economist.

Ludwig Otto Hesse (1811–1874) was a German mathematician.

Cornelius (Cornel) Lanczos (1893–1974) was a Hungarian mathematician and physicist.

Léon Nicolas Brillouin (1889–1969) was a French physicist.

George Green (1793–1841) was a British mathematical physicist.

Erwin Rudolf Josef Alexander Schrödinger (1887–1961) was a Nobel Prize-winning (1933) Austrian physicist who developed a number of fundamental results, which formed the basis of wave mechanics.

Sir William Rowan Hamilton (1805–1865) was an Irish physicist, astronomer and mathematician.

Paul Adrien Maurice Dirac (1902–1984) was an English theoretical physicist who made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics. He shared the Nobel Prize in physics for 1933 with Erwin Schrödinger "for the discovery of new productive forms of atomic theory".

Abraham de Moivre (1667–1754) was a French mathematician.

Giacinto Morera (1856–1909) was an Italian engineer and mathematician.

Baron Augustin-Louis Cauchy (1789–1857) was a French mathematician widely reputed as a pioneer of analysis.

Pierre-Simon, marquis de Laplace (1749–1827) was an influential French scholar whose work was important to the development of mathematics, statistics, physics and astronomy.

Leonhard Euler (1707–1783) was a pioneering Swiss mathematician and physicist.

Sir Isaac Newton (1642–1726/1727) was a famous English physicist and mathematician who laid the foundations for classical mechanics and made seminal contributions to optics and (together with Gottfried Leibniz) the development of calculus.

Gottfried Wilhelm von Leibniz (1646–1716) was a German polymath and philosopher, who to this day occupies a prominent place in the history of mathematics and the history of philosophy. Most scholars believe Leibniz developed calculus independently of Isaac Newton, and Leibniz's notation has been widely used ever since it was published.

Karl Theodor Wilhelm Weierstrass (1815–1897) was a German mathematician often cited as the "father of modern analysis".

Niels Henrik Abel (1802–1829) was a Norwegian mathematician.

Brook Taylor (1685–1731) was an English mathematician.

Pierre Alphonse Laurent (1813–1854) was a French mathematician.

Julian Karol Sochocki (1842–1927) was a Polish mathematician.

Felice Casorati (1835–1890) was an Italian mathematician.

Marie Ennemond Camille Jordan (1838–1922) was a French mathematician.

Hendrik Anthony Kramers (1894–1952) was a Dutch physicist.

Ralph Kronig (1904–1995) was a German-American physicist.

Max Karl Ernst Ludwig Planck (1858–1947) was a German theoretical physicist and one of the founders of the quantum theory.

Gerd Binnig (born 1947) is a German physicist.

Heinrich Rohrer (born 1933) is a Swiss physicist.

Julian Karol Sokhotski (1842–1927) was a Russian-Polish mathematician.

Josip Plemelj (1873–1967) was a Slovene mathematician.

Oliver Heaviside (1850–1925) was a self-taught English electrical engineer, mathematician and physicist.

Johann Carl Friedrich Gauss (1777–1855) was a German mathematician and physicist.

Lorenzo Mascheroni (1750–1800) was an Italian mathematician.

Benjamin Olinde Rodrigues (1795–1851), more commonly known as Olinde Rodrigues, was a French banker, mathematician and social reformer.

Carl Gustav Jacob Jacobi (1804–1851) was a German mathematician.

Friedrich Wilhelm Bessel (1784–1846) was a German astronomer, mathematician, physicist and geodesist.

James Stirling (1692–1770) was a Scottish mathematician.

William Lawrence Bragg (1890–1971) was an Australian-born British physicist and X-ray crystallographer, discoverer (1912) of the Bragg law of X-ray diffraction and a joint winner (with his father, Sir William Bragg) of the Nobel Prize for physics in 1915.

Niels Henrik David Bohr (1885–1962) was a Danish physicist who made fundamental contributions to quantum theory. He received the Nobel Prize in physics in 1922.

Ludwig Eduard Boltzmann (1844–1906) was an Austrian physicist and philosopher and one of the founders of statistical mechanics.

Ralph Kronig (1904–1995) was a German physicist.

William George Penney (1909–1991) was an English mathematician and mathematical physicist.

Felix Bloch (1905–1983) was a Swiss physicist and was awarded the 1952 Nobel Prize in physics.

Johann Peter Gustav Lejeune Dirichlet (1805–1859) was a German mathematician with deep contributions to number theory, the theory of Fourier series and other topics in mathematical analysis.

Marc-Antoine Parseval des Chênes (1755–1836) was a French mathematician.

Michel Plancherel (1885–1967) was a Swiss mathematician.

Takeo Matsubara (born 1921) is a Japanese theoretical physicist.

Siméon Denis Poisson (1781–1840) was a French mathematician, geometer and physicist.

Paul Peter Ewald (1888–1985) was a German-born US crystallographer and physicist and a pioneer of X-ray diffraction methods.

Max Born (1882–1970) was a German-British physicist and mathematician.

Theodore von Kármán (1881–1963) was a Hungarian-American mathematician, aerospace engineer and physicist.

Léon Nicolas Brillouin (1889–1969) was a French physicist.

Jean-Baptiste le Rond d'Alembert (1717–1783) was a French mathematician, mechanician, physicist, philosopher and music theorist.

Hermann Ludwig Ferdinand von Helmholtz (1821–1894) was a German physician and physicist.

Richard Phillips Feynman (1918–1988) was an American theoretical physicist who made several fundamental contributions in physics. He received the Nobel Prize in physics in 1965.

Named after Robert Brown (1773–1858) was a Scottish botanist and palaeobotanist.

Norbert Wiener (1894–1964) was an American mathematician and philosopher.

Aleksandr Yakovlevich Khinchin (1894–1959) was a Soviet mathematician.

Joseph Fraunhofer (1787–1826), ennobled in 1824 as Ritter von Fraunhofer, was a German optician.

Pierre-Simon, marquis de Laplace (1749–1827) was a French mathematician and astronomer.

Gustav Robert Kirchhoff (1824–1887) was a German physicist.

Josiah Willard Gibbs (1839–1903) was an American scientist who made important theoretical contributions to physics, chemistry and mathematics.

James Clerk Maxwell (1831–1879) was a Scottish scientist in the field of mathematical physics. His most notable achievement was to formulate the unified classical theory of electromagnetic radiation, bringing together for the first time electricity and magnetism.

Pythagoras of Samos (c. 570–c. 495 BC) was an Ionian Greek philosopher and mathematician.

Mikhail Vasilyevich Ostrogradsky (1801–1862) was a Ukrainian mathematician and physicist.

Joseph-Louis Lagrange (1736–1813) was an Italian Enlightenment Era mathematician and astronomer who made significant contributions to the fields of analysis, number theory and both classical and celestial mechanics.

Llewellyn Hilleth Thomas (1903–1992) was a British physicist and applied mathematician.

Enrico Fermi (1901–1954) was an outstanding Italian physicist and the 1938 Nobel laureate in physics.

Walter Kohn (born in 1923) is an Austrian-born American theoretical physicist. He was awarded, with John Pople, the Nobel Prize in chemistry in 1998 for the development of the density functional theory.

Lu Jeu Sham (born April 28, 1938) is a Chinese physicist.

Vladimir Aleksandrovich Fock (1898–1974) was a Soviet physicist.

Douglas Rayner Hartree (1897–1958) was an English mathematician and physicist.

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