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Stability Analysis of Nonlinear Systems

Second Edition

Birkhäuser

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PREFACE TO THE SECOND EDITION

The first edition of this book was published in 1989 by Marcel Dekker, Inc. For the last two decades the methods of stability investigation of nonlinear systems presented in this book have acquired further development and have been widely applied in the investigation of some types of contemporary dynamical systems. We note here the following directions only: practical stability; fuzzy differential equations and inclusions; set differential equations in metric spaces; fractional dynamic systems; causal differential equations; differential equations in cones; uncertain dynamical systems; dynamic equations on time scales; weakly connected nonlinear systems.

Since then this book became a rare one, meanwhile the methods of stability analysis set out in it possess a considerable potential for further development of qualitative theory of equations.

The present edition constitutes some expansion of the First Edition of this book. Namely, each of the Chapters 1–4 is added with new results displaying the development of stability theory. Chapter 5 is new. It contains applications of general methods for solution of applied problems. In particular, new stability conditions are presented for motion of a robot interacting with a dynamic medium. Conditions for stability are established for affine systems. Stability conditions in a physical system are indicated for synchronization of beams of two connected lasers. Stability conditions are found for the Takagi-Sugeno system and their application for impulse control in the "predator-prey" system is shown. These results have been obtained for the recent years at the Stability of Processes Department of the S.P. Timoshenko Institute of Mechanics of NAS of Ukraine.

The List of References at the end of the book is supplied with new citations of the authors papers published after 1989. This new list of references will enable all potential readers to approach the boundary of the investigations beyond which there are open problems in the fields.

The book is intended for graduate students and researchers in the applied mathematics, the physical sciences and any of the engineering disciplines, who are interested in the qualitative theory of equations. The authors strongly hope that any “2-places” in the book will be revealed by potential readers and each author would appreciate receiving any comments from scientific community.

We undertook our work over the second edition of the book in 2009. To our deepest regret the work was darkened by the death of Professor V.Lakshmikantham who passed away in 2012 and it is only now when we are completing the second edition.

We appreciate the efforts and patience of many colleagues at the Stability of Processes Department of the S.P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine in the preparing and correcting the manuscript.

Geneseo-Kiev, June 2015

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PREFACE

The problems of modern society are both complex and interdisciplinary. Despite the apparent diversity of problems, however, often tools developed in one context are adaptable to an entirely different situation. For example, consider Lyapynov's second method. This interesting and fruitful technique has gained increasing significance and has given decisive impetus for modern development of stability theory of differential equations. A manifest advantage of this method is that is does not require the knowledge of solutions and therefore has great power in applications. There are several books available expounding the main ideas of Lyapynov's second method, including some extensions and generalizations.

It is now recognized that the concept of Lyapynov-like function and the theory of differential and integral inequalities can be utilized to study qualitative and quantitative properties of non-linear differential equations. Lyapunov-like function serves as a vehicle to transform a given complicated differential system and therefore it is enough to investigate the properties of this simpler differential system. It is also being realized that the same versatile tools are adaptable to discuss entirely different nonlinear systems, and other tools such as the method of variation of parameters and monotone iterative technique provide equally effective methods to investigate problems of similar nature. Moreover, interesting new notions and ideas have been introduced which seem to possess great potential. Due to the increased interdependency and cooperation among mathematical sciences across the traditional boundaries and the accomplishments thus far achieved, there is every reason to believe that many breakthroughs are there waiting and offering an exiting prospect for this versatile technique to advance further.

It is in this spirit that wee see the importance of our monograph.

Its aim is to present a systematic account of the recent developments, describe the current state of the useful theory, show the essential unity achieved by the wealth of applications and provide a unified general structure applicable to a variety of nonlinear problems.

Some important features of the monograph are as follows: This is the first book that (i) presents a systematic study of stability theory in terms of two different measures and exhibits the advantage of the employing families of Lyapunov functions; (ii) treats the theory of a variety of inequalities clearly bringing out the underlying theme; and (iii) demonstrates the manifestations of general Lyapunov method by showing how this effective technique can be adapted to study various apparently diverse nonlinear problems.

This book also stresses the importance of utilizing different forms of nonlinear variations of parametric formulae to discuss qualitative behaviors of nonlinear problems, examines the constructive methods generated by monotone iterative technique and the method of upper and lower solutions, and illustrates the application of theoretical results to several different models chosen from real-world phenomena for the benefit of the practitioners.

In view of the existence of several excellent books on stability by Lyapunov's second method, we have restricted ourselves to presenting new developments, illustrative examples and useful applications, and consequently there is a minimum of overlap with the existing books.

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