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Ionela Prodan · Florin Stoican
Sorin Olaru · Silviu-Iulian Niculescu

Mixed-Integer Representations in Control Design

Mathematical Foundations and Applications

Springer

Ionela Prodan
Laboratory of Conception and Integration of Systems
Université Grenoble Alpes
Valence
France

Sorin Olaru
Laboratory of Signals and Systems
CentraleSupélec - CNRS - Université Paris-Sud, Université Paris-Saclay
Gif-sur-Yvette
France

Florin Stoican
Department of Automatic Control and Systems Engineering
Politehnica University of Bucharest
Bucharest
Romania

Silviu-Iulian Niculescu
Laboratory of Signals and Systems
CNRS - CentraleSupélec - Université Paris-Sud, Université Paris-Saclay
Gif-sur-Yvette
France

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Preface

“To be or not to be?”

(Hamlet, Shakespeare, 1601)

While the scope of a book is to present the problem discussed as esoterically as possible we also have to acknowledge its roots in “real life.” This is the case here with “control problems with contradictory conditions” which are readily found in everyday life. They may not be stated so, but each time we balance between mutually exclusive goals or have to choose a direction in the detriment of another we actually solve (optimally or not) such a control problem. An “either/or” decision is always difficult for human beings, the same goes for optimization algorithms with discrete variables. We have thus strong reasons to pursue solutions for this class of problems. As is usually the case, the solution is better or faster if we understand the problem’s underlying structure. Therefore, the scope of this book is to provide efficient constructions which are subsequently put under a mixed-integer form which can be solved by a computer in a “reasonable” time.

This book represents the culmination of over 5 years of collaboration work of the authors. The main contributions are the result of work started during the Ph.D. theses of the first two authors and by subsequent advancements in the following years.

This book was inspired by our desire to bring to light the importance of the analysis and control of dynamical systems with conflicting objectives and the effective usage of the associated mixed-integer formulations. It is worth mentioning that the topic is not new and monographs exists covering mixed-integer optimization. However, most of them assume a specific background in mathematics and optimization. The present book is dedicated to a generic class of constraints and their use in optimization problems and, in this respect, goes deeper into the details of their construction, representation and computational complexity. It is important to mention that the present manuscript is mainly dedicated to the problem description and not to the numerical optimization routines. It focuses on the mixed-integer aspects of the constraints formulation and their relationship with the optimization-based control design. To our knowledge, another textbook is not currently available that covers a compact treatment of the non-convex feasible set representation via mixed-integer representations, gathering the recent research advancements in the literature and illustrating the potential impact on optimization-based design, as for example in control design.

One of the most important features of the book is that it provides all along the manuscript the tools for easy reconstruction of the illustrative examples. The applications encompass important issues from control theory, ranging from motion planning with obstacle and collision avoidance and up to fault tolerant control schemes.

The book will hopefully not only serve the purpose of disseminating research results but also of raising the awareness for these challenging, timely and relevant research topics on optimization and control design. Moreover, we hope that this book will find attention in the diverse control engineering, computational mathematics and optimization communities and thus will contribute to the development of mixed-integer representations as a well-defined research field.

Valence	Ionela Prodan
Bucharest	Florin Stoican
Gif-sur-Yvette	Sorin Olaru
Gif-sur-Yvette	Silviu-Iulian Niculescu
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Contents

Abbreviations

FDI	Fault Detection and Isolation
FTC	Fault Tolerant Control
KKT	Karas–Kuhn–Tucker
LMI	Linear Matrix Inequalities
LP	Linear Programming
LQ	Linear Quadratic
LTI	Linear Time Invariant
MILCP	Mixed-Integer Linear Constrained Programming
MILP	Mixed-Integer Linear Programming
MINLP	Mixed-Integer Non-Linear Programming
MIP	Mixed-Integer Programming
MIQCP	Mixed-Integer Quadratically Constrained Programming
MIQP	Mixed-Integer Quadratic Programming
MPC	Model Predictive Control
NP-hard	Non-deterministic Polynomial-time hard
PWA	Piecewise Affine
QP	Quadratic Programming
RC	Reconfiguration Control
RPI	Robust Positive Invariance
UAV	Unmanned Aerial Vehicle

Notation

The conventions and the notations used in the book are the ones classically employed in control theory literature.

- $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{N}$ denote the set of real numbers, the set of integers and the set of non-negative integers, respectively.
- $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the vector field with n elements and the real matrices field with n rows and m columns, respectively. The same notation is adopted for the sets of integer and non-negative integers.
- I denotes the identity matrix of appropriate dimension and e_i ($e_i^\top$) denotes its ith column (row).
- **1** denotes the matrix of ones and **0** the matrix of zeros of appropriate dimension.
- For a matrix $A \in \mathbb{R}^{n\times n}$, the standard notation $A^\top$ denotes the transpose of matrix A, A^{-1} denotes the inverse of matrix A and $A \succ 0$ ($A \succeq 0$) denotes a (strictly) positive definite matrix.
- $||z||_M$ is the weighted Euclidean norm, i.e., $\sqrt{(z^{\mathrm{T}}Mz)}$.
- For a discrete-time signal $x \in \mathbb{R}^n$, the current and successor states are denoted as $x(k)$ and $x(k+1)$, respectively.
- Absolute values and vector inequalities are considered elementwise (unless otherwise explicitly stated), that is, $|T|$ denotes the elementwise magnitude of a matrix T and $x \leq y$ ($x < y$) denotes the set of elementwise (strict) inequalities between the components of the real vectors x and y.
- The ceiling value of $x \in \mathbb{R}$, denoted by $\lceil \mathrm{x} \rceil$, is the smallest integer greater than x.

For a given set $S \in \mathbb{R}^n$:

- $\overline{s} = \max\limits_{s\in S} s$ denotes the elementwise maximum where each element is computed as $\overline{s}_i = \max\limits_{s\in S} s_i$. The elementwise minimum, $\underline{s}$ is defined similarly.
- For a matrix $A \in \mathbb{R}^{n\times m}$ we define the set $AS = \{z \in \mathbb{R}^n : z = Ax, \forall x \in S\}$.
- $\overline{S}$ denotes the complement of the set S which refers to elements not in (that is, elements outside of) the set S.

- cl(S) denotes the closure of set S which is defined as the union of S and its boundary.
- card(S) denotes the cardinality of a set S which is a measure of the number of elements of the set.
- $\mathrm{Cone}(x, S) = \{x + \alpha(x - s), \forall s \in S, \forall \alpha \geq 0\}$ denotes the pointed cone with extreme point x and tangent to set S.

For given sets $X, Y \in \mathbb{R}^n$:

- $\mathrm{Conv}(X, Y) = \{\alpha x + (1 - \alpha)y, \forall x \in X, \forall y \in Y, 0 \leq \alpha \leq 1\}$ is the convex hull of the sets X and Y.
- $X \cap Y$ denotes the set intersection between X and Y.
- $X \subset (\subseteq) Y$ denotes that X is a (strict) subset of Y.
- $X \oplus Y = x + y : x \in X,\ y \in Y$ defines the Minkowski addition of sets X and Y.
- $X \ominus Y = x \in X : x \oplus Y \subseteq X$ defines the Pontryagin difference of sets X and Y.
- The collection of all possible N combinations of binary variables is denoted by: $\{0, 1\}^N = \{(b_1 \ldots b_N) : b_i\{0, 1\}, i = 1 \ldots N\}$.
  The same definition holds for sign tuples $\{-, +\}^N$.
- For a binary variable f with values in $\{0, 1\}$, notation $\bar{f}$ denotes $\bar{f} = 1 - f$. The same holds for $f \in \{-, +\}$ where $\bar{f} = `-'$ if $f = `+'$ and $\bar{f} = `+'$ if $f = `-'$.
- $\mathrm{lp}(n, d)$ and $\mathrm{qp}(n, d)$ represent the complexity of solving a linear program, quadratic program respectively, with n constraints and d variables.