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Kasia Rejzner

Perturbative Algebraic Quantum Field Theory

An Introduction for Mathematicians

 Springer

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*This book is dedicated to the memory
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and John Roberts, who passed
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