Geospatial Algebraic Computations

Joseph L. Awange • Béla Paláncz

Geospatial Algebraic Computations

Theory and Applications

Third Edition

Springer

Joseph L. Awange
Curtin University
Perth, West Australia
Australia

Béla Paláncz
Budapest University of Technology
and Economics
Budapest, Hungary

Additional material to this book can be downloaded from http://extras.springer.com.

ISBN 978-3-319-25463-0 ISBN 978-3-319-25465-4 (eBook)
DOI 10.1007/978-3-319-25465-4

Library of Congress Control Number: 2015958020

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer-Verlag GmbH Berlin Heidelberg is part of Springer Science+Business Media (www.springer.com)

Foreword

I compliment the authors in this book because it brings together mathematical methods for the solution of multivariable polynomial equations that hardly are covered side by side in any ordinary mathematical book: The book explains both *algebraic* (*exact*) methods and *numerical* (*approximate*) methods. It also points to the recent combination of algebraic and numerical methods (*hybrid* methods), which is currently one of the most promising directions in the area of computer mathematics. The reason why this book manages to bring the algebraic and the numerical aspect together is because it is strictly goal oriented toward the solution of fundamental problems in the area of geodesy and geoinformatics – e.g., the positioning problem – and the solution of application problems does not allow purism in methodology but, rather, has to embrace different approaches with different benefits in different circumstances.

Personally, it is very fulfilling for me to see that my *Groebner bases* methodology, mainly by the work of the authors, finds now also useful applications in the area of geodesy and geoinformatics. Since the book compares, in the applications, *Groebner bases* and *resultants* as the two main algebraic approaches, it also gives a lot of new motivations for further mathematical research in the relationship between these two approaches, which is still not well understood.

All good wishes for the further success of this book in the community of both *geoinformatics* and *computer mathematics*!

Hagenberg, Austria
September 2009

Prof. Dr.phil. Dr.h.c.mult. Bruno Buchberger
Professor of Computer Mathematics and
Head of Softwarepark Hagenberg

Preface to the First Edition

While preparing and teaching "Introduction to Geodesy I and II" to undergraduate students at Stuttgart University, we noticed a gap which motivated the writing of the present book: Almost every topic that we taught required some skills in algebra and, in particular, computer algebra! From positioning to transformation problems inherent in geodesy and geoinformatics, knowledge of algebra, and application of computer algebra software were required. In preparing this book therefore, we have attempted to put together basic concepts of *abstract algebra* which underpin the techniques for solving algebraic problems. Algebraic computational algorithms useful for solving problems which require exact solutions to nonlinear systems of equations are presented and tested on various problems. Though the present book focuses mainly on the two fields, the concepts and techniques presented herein are nonetheless applicable to other fields where algebraic computational problems might be encountered. In engineering for example, network densification and robotics apply resection and intersection techniques which require algebraic solutions.

Solution of nonlinear systems of equations is an indispensable task in almost all geosciences such as geodesy, geoinformatics, and geophysics (just to mention but a few) as well as robotics. These equations which require exact solutions underpin the operations of ranging, resection, intersection, and other techniques that are normally used. Examples of problems that require exact solutions include:

- three-dimensional resection problem for determining positions and orientation of sensors, e.g., camera, theodolites, robots, scanners, etc.,
- coordinate transformation to match shapes and sizes of points in different systems,
- mapping from topography to reference ellipsoid,
- analytical determination of refraction angles in GPS meteorology.

The difficulty in solving explicitly these nonlinear systems of equations has led practitioners and researchers to adopt approximate numerical procedures, which often have to do with linearization, approximate starting values, and iterations, and sometimes require a lot of computational time. In order to offer solutions to the challenges posed by nonlinear systems of equations, this book provides, in a pioneering work, the application of *ring* and *polynomial theories*, *Groebner basis*, *polynomial resultants*, *Gauss-Jacobi combinatorial*, and *Procrustes algorithms*. Users faced with algebraic computational problems are thus provided with algebraic tools that are not only a must but essential and have been out of reach. For these users, most algebraic books at their disposal have unfortunately been written in mathematical formulations suitable to mathematicians. We strive to simplify the algebraic notions and provide examples where necessary to enhance easier understanding.

For those in mathematical fields such as applied algebra, symbolic computations and application of mathematics to geosciences, etc., the book provides some practical examples of application of mathematical concepts. Several geodetic and geoinformatic problems are solved in the book using methods of abstract algebra and multidimensional scaling. These examples might be of interest to some mathematicians.

Chapter 1 introduces the book and provides a general outlook on the main challenges that call for algebraic computational approaches. It is a motivation for those who would wish to perform analytical solutions. Chapter 2 presents the basic concepts of ring theory relevant for those readers who are unfamiliar with abstract algebra and therefore prepare them for latter chapters which require knowledge of ring axioms. Number concept from operational point of view is presented. It is illustrated how the various sets of natural numbers $\mathbb{N}$, integers $\mathbb{Z}$, quotients $\mathbb{Q}$, real numbers $\mathbb{R}$, complex numbers $\mathbb{C}$, and quaternions $\mathbb{H}$ are vital for daily operations. The chapter then presents the concept of ring theory. Chapter 3 looks at the basics of polynomial theory, the main object used by the algebraic algorithms that will be discussed in the book. The basics of polynomials are recaptured for readers who wish to refresh their memory on the basics of algebraic operations. Starting with the definition of polynomials, Chap. 3 expounds on the concept of polynomial rings, thus linking it to the number ring theory presented in Chap. 2. Indeed, the theorem developed in the chapter enables the solution of nonlinear systems of equations that can be converted into (algebraic) polynomials.

Having presented the basics in Chaps. 2 and 3, Chaps. 4, 5, 6, and 7 present algorithms which offer algebraic solutions to nonlinear systems of equations. They present theories of the procedures starting with the basic concepts and showing how they are developed to algorithms for solving different problems. Chapters 4, 5, and 6 are based on *polynomial ring theory* and offer an in-depth look at the basics of *Groebner basis*, *polynomial resultants*, and *Gauss-Jacobi combinatorial algorithms*. Using these algorithms, users can develop their own codes to solve problems requiring exact solutions.

In Chap. 7, the Global Positioning Systems (GPS) and the Local Positioning Systems (LPS) that form the operational basis are presented. The concepts of local datum choice of types $\mathbb{E}^*$ and $\mathbb{F}^*$ are elaborated, and the relationship between local reference frame $\mathbb{F}^*$ and the global reference frame $\mathbb{F}^\bullet$, together with the resulting observational equations, is presented. The test network "Stuttgart Central" in Germany that we use to test the algorithms in Chaps. 4, 5, and 6 is also presented in this chapter. Chapter 8 deviates from the polynomial approaches to present a linear algebraic (analytical) approach of Procrustes that has found application in fields such as *medicine* for gene recognition and *sociology* for crime mapping. The chapter presents only the partial Procrustes algorithm. The technique is presented as an efficient tool for solving algebraically *the three-dimensional orientation problem* and the *determination of vertical deflection*.

From Chaps. 9 to 15, various computational problems of algebraic nature are solved. Chapter 9 looks at the ranging problem and considers both the GPS pseudo-range observations and ranging within the LPS systems, e.g., using EDMs. The chapter presents a complete algebraic solution starting with the simple planar case to the three-dimensional ranging in both closed and overdetermined forms. Critical conditions where the solutions fail are also presented. Chapter 10 considers the Gauss ellipsoidal coordinates and applies the algebraic technique of Groebner basis to map topographic points onto the reference ellipsoid. The example based on the Baltic Sea level project is presented. Chapters 11 and 12 consider the problems of resection and intersection, respectively.

Chapter 13 discusses a modern and relatively new area in Geodesy, the GPS meteorology. The chapter presents the theory of GPS meteorology and discusses both the space-borne and ground- based types of GPS meteorology. The ability of applying the algebraic techniques to derive refraction angles from GPS signals is presented. Chapter 14 presents an algebraic deterministic version to outlier problem, thus deviating from the statistical approaches that have been widely publicized. Chapter 15 introduces the seven-parameter datum transformation problem commonly encountered in practice and presents the general Procrustes algorithm. Since this is an extension of the partial Procrustes algorithm presented in Chap. 8, it is referred to as Procrustes algorithm II. The chapter further presents an algebraic solution of the transformation problem using Groebner basis and Gauss-Jacobi combinatorial algorithms. The book is completed in Chap. 16 by presenting an overview of modern computer algebra systems that may be of use to geodesists and geoinformatists.

Many thanks to Prof. B. Buchberger for his positive comments on our Groebner basis solutions; Prof. D. Manocha who discussed the resultant approach; Prof. D. Cox who also provided much insight in his two books on rings, fields, and algebraic geometry; and Prof. W. Keller of Stuttgart University Germany, whose door was always open for discussions. We sincerely thank Dr. J. Skidmore for granting us permission to use the Procrustes "magic bed" and related materials from Mythweb.com. Thanks to Dr. J. Smith (editor of Survey Review), Dr. S. J. Gordon, and Dr. D. D. Lichti for granting us permission to use the scanner resection figures appearing in Chap. 12. We are also grateful to Chapman and Hall Press for granting us permission to use Fig. 9.2 where malarial parasites are identified using Procrustes. Special thanks to Prof. I. L. Dryden for permitting us to refer to his work and all the help. Many thanks to Ms F. Wild for preparing Figs. 16.9 and 17.7. We acknowledge your efforts and valuable time. Special thanks to Prof. A. Kleusberg of Stuttgart University Germany; Prof. T. Tsuda of Radio Center for Space and Atmosphere, Kyoto University Japan; Dr. J. Wickert of GeoForschungsZentrum Potsdam (GFZ), Germany; and Dr. A. Steiner of the Institute of Meteorology and Geophysics, University of Graz, Austria, for the support in terms of literature and discussions on Chap. 18. The data used in Chap. 13 were provided by GeoForschungsZentrum Potsdam (GFZ). For these, the authors express their utmost appreciation.

The first author also wishes to express his utmost sincere thanks to Prof. S. Takemoto and Prof. Y. Fukuda of the Department of Geophysics, Kyoto University, Japan, for hosting him during the period of September 2002 to September 2004. In particular Chap. 13 was prepared under the supervision and guidance of Prof. Y. Fukuda: Your ideas, suggestions, and motivation enriched the book. For these, we say *arigato gozaimashita* – Japanese equivalent to *thank you very much*. The first author's stay at Kyoto University was supported by Japan Society of Promotion of Science (JSPS): The author is very grateful for this support. The first author is grateful to his wife Mrs. *Naomi Awange* and his two daughters *Lucy* and *Ruth* who always brightened him up with their cheerful faces. Your support, especially family time that I denied you in order to prepare this book, is greatly acknowledged. Naomi, thanks for carefully reading the book and correcting typographical errors. However, the authors take full responsibility of any typographical error. Last but not least, the second author wants to thank his wife *Ulrike Grafarend*, his daughter *Birgit*, and his son *Jens* for all the support over these many years as they were following him at various places around the globe.

Kyoto, Japan and Stuttgart, Germany — Joseph L. Awange
September 2004 — Erik W. Grafarend

Preface to the Second Edition

This work is in essence the second edition of the 2005 book by Awange and Grafarend Solving Algebraic Computational Problems in Geodesy and Geoinformatics. This edition represents a major expansion of the original work in several ways.

Realizing the great size of some realistic *geodetic* and *geoinformatic* problems that cannot be solved by pure *symbolic algebraic* methods, combinations of the *symbolic and numeric techniques*, so-called *hybrid techniques*, have been introduced. As a result, new chapters have been incorporated to cover such numeric methods. These are *Linear Homotopy* in Chap. 6 and the *Extended Newton-Raphson* in Chap. 8, with each chapter accompanied by new numerical examples. Other chapters dealing with the basics of polynomial theory, LPS-GPS orientations, and vertical deflections, as well as GNSS meteorology in environmental monitoring, have been refined. We also point out that computer algebra system (Chap. 16) of the first edition has been omitted in the present book due to the rapid changing of computational algorithms.

In the meantime, since the date of the first edition, some earlier methods have been improved. Therefore, chapters like Procrustes Solution and Datum Transformation Problems have been expanded and the associated improvements in the symbolic-numeric methods are demonstrated for the case of affine transformation with nine parameters.

In order to emphasize the theoretical background of the methods and their practical applications to geodetic and geoinformatic problems, as well as to compare and qualify them for different applications, the book has been split into two parts. Part I covers the theoretical concepts of the algebraic symbolic and numeric methods, and as such, readers already familiar with these can straight away move to the applications covered in Part II of the book. Indeed, Part II provides in-depth practical applications in geodesy and geoinformatics.

Perhaps the most considerable extension from a *theoretical* as well as from a *practical* point of view is the electronic supplement to the book in CD form. This CD contains 20 chapters and about 50 problems solved with different *symbolic*, *numeric*, and *hybrid* techniques using one of the leading computer algebraic systems CAS's *Mathematica*. The notebooks provide the possibility of carrying out real-time computations with different data or models. In addition, some Mathematica modules representing algorithms discussed in the book are supplied to make it easier for the reader to solve his/her own real geodetic/geoinformatic problems. These modules are open source; therefore they can be easily modified by users to suit their own special purposes. The effectiveness of the different methods is compared and qualified for different problems and some practical recipes given for the choice of the appropriate method. The actual evaluation of the codes as parallel computation on multi-core/processor machines is also demonstrated. For users not familiar with the Mathematica system, the pdf versions of the notebooks are also provided.

Last, but not least, the company of the authors has also been extended, demonstrating that nowadays the cooperation of peoples from different scientific fields is indispensable when writing such a comprehensive book.

Overall, in this second edition, we have tried to bring together the *basic theories* and their geodesic/geoinformatic applications as well as the practical realization of these algorithms. In addition, the extensive references listing should help interested readers to immerse themselves in the different topics more deeply.

We have attempted to correct the various errors that were inadvertently left in the first edition; however readers are encouraged to contact us about errors or omissions in the current edition.

Many thanks go to *Prof. B. Buchberger*, the *father* of *Groebner basis method*, for his positive comments on our Groebner basis solutions and for agreeing to write a foreword for our book and to *Prof. R. Lewis* for explaining his EDF method (Early Discovery Factor) to compute Dixon resultant as well as for carrying out some computations with his algebraic computer code Fermat. We are also grateful to *Dr. D. Lichtblau* for helping us to learn the proper and efficient use of Mathematica, especially the Groebner basis algorithm as well as, to write some appreciating words for the back cover of the book. Special thanks to Dr. *K. Flemming* and Dr. *C. Hirt* of Curtin University for sparing time to proofread this edition and for providing valuable comments. Further thanks to *K. Flemming* for preparing Fig. 20.4.

J.L. Awange wishes to express his utmost sincere thanks to *Prof. B. Heck* of the Department of Physical Geodesy (Karlsruhe University, Germany) for hosting him during the period of the Alexander von Humboldt Fellowship (2008–2011). In particular, your ideas, suggestions, and motivations on Chap. 6 enriched the book. *J.L. Awange* is further grateful to *Prof. B. Veenendaal* (HoD, Spatial Sciences, Curtin University of Technology, Australia) and *Prof. F. N. Onyango* (Vice Chancellor, Maseno University, Kenya) for the support and motivation that enabled the preparation of this edition. Last, but not least, *J.L. Awange's* stay at Curtin University of Technology is supported by *Curtin Research Fellowship*, while his stay at Karlsruhe University is supported by *Alexander von Humboldt's Ludwig Leichhardt's Memorial Fellowship*: The author is very grateful for this financial support.

Perth, Australia, Stuttgart, Germany,
and Budapest, Hungary
October 2009

Joseph L. Awange
Erik W. Grafarend
Béla Paláncz
Piroska Zaletnyik

Preface to the Third Edition

This third edition that includes a completely revised form of the techniques presented in the previous editions introduces three new symbolic-numeric methods that have started to spread in geospatial sciences. The employment of these efficient methods is required mainly by the wide-spreading application of laser techniques producing huge cloud of data, hence necessitating a change of title from Algebraic Geodesy and Geoinformatics in the second edition to Geospatial Algebraic Computation in the current edition. In the first part of the book, these three new Chaps. 10, 11, and 12 (Pareto Optimality of Multi-objective Optimization, Symbolic Regression, and Robust Estimation) represent the increasing importance of intelligent data analysis and the modern handling of large data sets. In addition, the linear homotopy chapter in the second edition is extended to incorporate the nonlinear homotopy.

In the second part of the book, the applications of these techniques are illustrated in practical geodetic problems, such as global and local positioning by ranging, resections, and intersections; datum transformation problems; GNSS environmental monitoring; detecting outliers; modeling local GPS/leveling geoid undulations; estimation of geometric primitives in LiDAR cloud of data; application of robust parameter estimation for GNSS data; and so on. The earlier topics as polynomials, Groebner basis, resultants, Gauss-Jacobi combinatorial and Procrustes algorithms, homotopy methods, as well as the new ones are illustrated by numerous practical geodetic examples in the form of fully explained notebooks created by the latest version of Mathematica in cloud computing environment representing the state of art of symbolic-numeric computation techniques. In addition these generally usable functions and packages are attached as the accompanying electronic material.

J.L. Awange wishes to express his sincere thanks to Prof. B. Heck (Karlsruhe Institute of Technology (KIT), Germany) for hosting him during the period of his Alexander von Humboldt Fellowship (June to September 2015), Prof. Y. Fukuda (Kyoto University, Japan) for hosting him during the period of his Japan Society of Promotion of Science (JSPS) Fellowship (October to December 2015), and Prof R. Goncalves of Federal University of Pernambuco (Brazil) for hosting him during his Science Without Boarder (January to March 2016). Parts of this book were written during these periods. He is also grateful to Prof. B. Veenendaal (head of Department, Spatial Sciences, Curtin University, Australia) for the support and motivation that enabled the preparation of this edition. He also wishes to acknowledge the support of Alexander von Humboldt that facilitated his stay at KIT, JSPS that supported his stay at Kyoto University, and Capes for supporting his stay in Brazil. To all, he says, “ahsante sana” (Swahili for thank you very much). Special thanks go to his family, namely, Mrs Naomi Awange, Lucy Awange, and Ruth Awange who had to contend with his long periods of absence from home. *B. Paláncz* expresses his high appreciation and thanks to Prof. Bert Veenendaal the head of the Department of Spatial Sciences (Curtin University, Australia) for his hospitality and financial support of his visiting Curtin. Béla Paláncz wishes also to thank the TIGeR Foundation for financing part of his stay at Curtin University, Perth. This is a TIGeR No. 633

Curtin University, Australia,
Karlsruhe Institute of Technology, Germany,
Kyoto University, Japan and
Federal University of Pernambuco, Brazil,
Budapest University of Technology and Economics Budapest, Hungary
July 2015

Joseph L. Awange
Béla Paláncz

Contents