

Springer Theses

Recognizing Outstanding Ph.D. Research

More information about this series at <http://www.springer.com/series/8790>

Aims and Scope

The series “Springer Theses” brings together a selection of the very best Ph.D. theses from around the world and across the physical sciences. Nominated and endorsed by two recognized specialists, each published volume has been selected for its scientific excellence and the high impact of its contents for the pertinent field of research. For greater accessibility to non-specialists, the published versions include an extended introduction, as well as a foreword by the student’s supervisor explaining the special relevance of the work for the field. As a whole, the series will provide a valuable resource both for newcomers to the research fields described, and for other scientists seeking detailed background information on special questions. Finally, it provides an accredited documentation of the valuable contributions made by today’s younger generation of scientists.

Theses are accepted into the series by invited nomination only and must fulfill all of the following criteria

- They must be written in good English.
- The topic should fall within the confines of Chemistry, Physics, Earth Sciences, Engineering and related interdisciplinary fields such as Materials, Nanoscience, Chemical Engineering, Complex Systems and Biophysics.
- The work reported in the thesis must represent a significant scientific advance.
- If the thesis includes previously published material, permission to reproduce this must be gained from the respective copyright holder.
- They must have been examined and passed during the 12 months prior to nomination.
- Each thesis should include a foreword by the supervisor outlining the significance of its content.
- The theses should have a clearly defined structure including an introduction accessible to scientists not expert in that particular field.

Mikhail P. Solon

Heavy WIMP Effective Theory

Formalism and Applications for Scattering
on Nucleon Targets

Doctoral Thesis accepted by The University of Chicago,
Chicago, Illinois, USA

Mikhail P. Solon
University of California, Berkeley
Berkeley, CA, USA

ISSN 2190-5053

ISSN 2190-5061 (electronic)

Springer Theses

ISBN 978-3-319-25197-4

ISBN 978-3-319-25199-8 (eBook)

DOI 10.1007/978-3-319-25199-8

Library of Congress Control Number: 2015953441

Springer Cham Heidelberg New York Dordrecht London

© Springer International Publishing Switzerland 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

To Orville and Sophia Solon

Supervisor's Foreword

Mikhail Solon's excellent thesis formulates the new heavy WIMP effective theory. This theory allows precise determination of WIMP-nucleon scattering cross sections in the absence of detailed knowledge about underlying dark sector dynamics, essential for planning and interpreting the results of future dark matter direct detection experiments.

Exploring the possible interactions between WIMPs and standard model particles has exposed gaps in our understanding of effective quantum field theory. Besides explicit computations of dark matter properties, Solon's thesis addresses several basic theoretical questions. It describes a new formalism for implementing Lorentz invariance constraints in nonrelativistic theories with unknown, or unspecified, ultraviolet completion. The thesis also provides new perturbative QCD results that determine the impact of heavy quarks on the hadronic matrix elements involved in WIMP direct detection.

Solon's thesis is a technical tour de force, with impacts in dark matter phenomenology, field theory formalism, and precision hadronic physics. His computation of scattering rates for heavy electroweak-charged dark matter has become a benchmark for the field of direct detection, and his thesis has also spawned a new field of investigation in dark matter indirect detection, determining heavy WIMP annihilation rates using effective field theory methods. The systematic treatment of renormalization and heavy quark threshold matching conditions, the review of background field methods, and the survey of hadronic matrix elements should be useful to students and researchers interested in a variety of new physics searches and precision measurements involving weak probes of the nucleon.

Chicago, IL, USA

Richard J. Hill

Preface

The discovery of a standard model-like Higgs boson and the hitherto absence of evidence for other new states may indicate that if WIMPs comprise cosmological dark matter, they are heavy compared to electroweak scale particles, $M \gg m_{W^\pm}, m_{Z^0}$. In this limit, the absolute cross section for a WIMP of given electroweak quantum numbers to scatter from a nucleon becomes computable in terms of standard model parameters. Extending aspects of heavy particle formalism familiar from heavy quark effective theory, we develop heavy WIMP effective theory to isolate universal behavior within the WIMP paradigm.

We present ingredients necessary for this effective theory framework, including the formalism for bottom-up construction of heavy particle Lagrangians based on induced representations of the Lorentz group, the complete calculation of one- and two-loop weak-scale matching amplitudes, a consistent renormalization scheme in the presence of nontrivial residual masses, and the QCD framework for passing from the theory renormalized at the electroweak scale to the theory of quarks and gluons below the charm scale.

We analyze the heavy WIMP limit of WIMP-nucleon scattering and present the first complete calculation of the leading spin-independent cross section in standard model extensions consisting of one or two electroweak $SU(2)_W \times U(1)_Y$ multiplets, including a careful treatment of perturbative and hadronic-input uncertainties. The standard model exhibits a surprising transparency of nucleons to WIMP scattering, due to a cancellation between scalar and tensor amplitude contributions. The resulting cross-section predictions and their fractional uncertainties depend sensitively on parameter inputs, and we investigate the impact of model-independent inputs, such as perturbative QCD corrections and nucleon scalar matrix elements, and of model-dependent inputs, such as WIMP quantum numbers, additional electroweak multiplets, and extended Higgs sectors.

Berkeley, CA, USA

Mikhail P. Solon

Acknowledgments

I lived the past 5 years with only one responsibility: to learn physics. I owe the pleasure of such a simple existence to a large number of people.

Foremost, I am indebted to my advisor Richard Hill. Working with him was a strong experience that showed me a high standard and the qualities for both doing good science and being a good person. Richard Hill is Richard Hill, and I consider it one of life's good fortunes to have been his student.

I enjoyed and learned a lot from collaborating with Johannes Heinonen, Richard Hill, Gabriel Lee, and Gil Paz. I am happy and proud of our work.

For many valuable discussions on physics, I would like to thank the members of my thesis committee as well as professors and postdocs of the theory group: Edward Blucher, Dam Son, Richard Hill, Carlos Wagner, Lian-Tao Wang, Jonathan Rosner, Brian Batell, Stefania Gori, and Johannes Heinonen.

The guidance and support of Stuart Gazes, Sandy Heinz, Autym Henderson, Nobuko McNeill, Beth Nakatsuka, and David Reid were essential to navigating graduate school.

I will always have good memories of Chicago, Slade, Jackson, Crown, and other wonderful places, due especially to the people I shared them with: Jeremy Berg, Pauline Baniqued, Denis Erkal, Szilard Farkas, Michael Fedderke, Simone Ferraro, Michael Geracie, Siavash Golkar, Stephen Green, Aniket Joglekar, Hridesh Kedia, Gabriel Lee, Jennifer Lin, Matthew Low, Travis Maxfield, Samuel Meehan, Marc Miskin, Dung Nguyen, Jason Pastrana, Kartik Prabhu, Callum Quigley, Joshua Schiffrin, Pronoy Sircar, Jordan Webster, and Brenton Wright.

I owe everything to Pauline, for her patience and love and for letting me hunker down in her apartment to write, and to my parents, Orville and Sophia, and my sister, Luna, for shaping who I am and what I can become.

I am deeply grateful for these people and many others.

This thesis was supported by a Bloomenthal Fellowship and presents results collected from the following of the author's works:

R. J. Hill and M. P. Solon, "Standard model anatomy of WIMP dark matter direct detection II: QCD analysis and hadronic matrix elements," to be published.

- R. J. Hill and M. P. Solon, “Standard model anatomy of WIMP dark matter direct detection I: weak-scale matching,” arXiv:1401.3339 [hep-ph].
- R. J. Hill and M. P. Solon, “WIMP-nucleon scattering with heavy WIMP effective theory,” Phys. Rev. Lett. **112**, 211602 (2014). [arXiv:1309.4092 [hep-ph]].
- R. J. Hill, G. Lee, G. Paz and M. P. Solon, “NRQED Lagrangian at order $1/M^4$,” Phys. Rev. D **87**, no. 5, 053017 (2013) [arXiv:1212.4508 [hep-ph]].
- J. Heinonen, R. J. Hill and M. P. Solon, “Lorentz invariance in heavy particle effective theories,” Phys. Rev. D **86**, 094020 (2012) [arXiv:1208.0601 [hep-ph]].
- R. J. Hill and M. P. Solon, “Universal behavior in the scattering of heavy, weakly interacting dark matter on nuclear targets,” Phys. Lett. B **707**, 539 (2012) [arXiv:1111.0016 [hep-ph]].

Contents

1	Heavy WIMP Effective Theory	1
1.1	Introduction	1
1.2	Universal Heavy WIMP Limit	4
1.3	Motivations for Heavy WIMP Effective Theory	7
1.4	Chapter Organization	9
2	Heavy-Particle Spacetime Symmetries and Building Blocks	13
2.1	Finite Dimensional Representations of the Lorentz Algebra	16
2.2	Effective Field Theory and the Little Group	18
2.2.1	Little Group Formalism	18
2.2.2	Field Transformation Law and Lorentz Invariance	20
2.2.3	$1/M$ Expansion and Lagrangian Constraints	22
2.3	Reparametrization Invariance and Invariant Operators	24
2.3.1	Covariant Notation	24
2.3.2	Reparametrization Invariance	26
2.3.3	Invariant Operator Method	26
2.3.4	Solution for $\Gamma(v, iD)$	27
2.4	Higher-Spin and Self-conjugate Fields	30
2.4.1	Higher Spin Representations	30
2.4.2	Self-conjugate Parity and CPT	32
2.5	NRQED Example: Lagrangian	33
2.6	NRQED Example: Relativistic Invariance	35
2.6.1	Variational Method	35
2.6.2	Invariant Operators	37
2.7	NRQED Example: One-Photon Matching	38
2.8	NRQED Example: Photon and Four-Fermion Sectors	40
2.8.1	Pure Photon Operators	40
2.8.2	Four-Fermion Operators	41
2.8.3	Field Redefinitions and Redundant Operators	43
2.8.4	Relativistic Lepton	45
2.9	Discussion	46

3	Effective Theory at the Weak-Scale	49
3.1	Singlet.....	50
3.1.1	Standard Model Building Blocks	50
3.1.2	Dark Matter Building Blocks	53
3.1.3	High-Energy Basis.....	53
3.1.4	Low-Energy Basis	55
3.2	Multiplets and Mixtures.....	57
3.2.1	Pure States	59
3.2.2	Higher-Order Example: Pure Triplet Scalar	61
3.2.3	Admixtures.....	63
3.2.4	Pure Case Limits.....	67
3.2.5	Relativistic Example: Singlet-Doublet Mixture	68
3.3	Onshell Renormalization Scheme	69
3.3.1	Singlet-Doublet Counterterm Lagrangian	69
3.3.2	Propagator Corrections	70
3.3.3	Renormalization Conditions	72
3.3.4	Extension to Triplet-Doublet	73
3.4	Low Energy Theory at the Weak Scale for Pure- and Mixed-State WIMPs.....	75
4	Weak-Scale Matching	77
4.1	Singlet.....	78
4.1.1	Case I: $M \lesssim m_b \ll m_W$	78
4.1.2	Case II: $m_W \lesssim M$	80
4.1.3	Case III: $m_W \ll M$	81
4.2	Multiplets and Mixtures.....	81
4.2.1	Quark Matching: One-Boson Exchange	82
4.2.2	Gluon Matching: One-Boson Exchange	86
4.2.3	Quark Matching: Two-Boson Exchange.....	87
4.2.4	Gluon Matching: Two-Boson Exchange.....	92
4.2.5	Effective Theory Amplitudes and Infrared Regulator.....	112
4.2.6	Extended Higgs Sector for Pure Case	113
4.2.7	Bare Matching Coefficients	114
5	QCD Analysis and Hadronic Matrix Elements	119
5.1	Operator Renormalization.....	120
5.1.1	Renormalization Constants.....	121
5.1.2	Renormalized Matching Coefficients for Pure States	123
5.2	Renormalization Group Evolution	125
5.3	Threshold Matching and Low Energy Coefficients	127
5.3.1	Heavy Quark Threshold Matching Conditions	128
5.3.2	Low Energy Coefficients	129
5.4	Hadronic Matrix Elements	130
5.4.1	Scalar Matrix Elements.....	130
5.4.2	Tensor Matrix Elements	132

6 Heavy WIMP-Nucleon Scattering Cross Sections 135

 6.1 Cross Section Assembly Line 136

 6.2 Survey of Uncertainties 138

 6.3 Cross Section Predictions and Consistency Checks 140

7 Conclusions 147

Appendix A: Solution to the Invariance Equation 151

 A.1 Series Solution for Γ 151

 A.2 Explicit Solution for Γ in the Spin 1/2 Theory 153

Appendix B: Integrals and Inputs for Weak Scale Matching 157

 B.1 Self Energy Integrals and Standard Model Two-Point
 Functions 157

 B.2 Box Integrals 161

 B.3 Heavy Particle Integrals with Electroweak Polarization
 Tensor Insertion 165

 B.3.1 Case of Zero Heavy Fermions 166

 B.3.2 Case of One Heavy Fermion 167

 B.3.3 Case of Two Heavy Fermions 170

 B.4 Numerical Inputs 170

**Appendix C: Inputs for Analysis of QCD Effects
and Hadronic Matrix Elements** 173

 C.1 QCD Functions 173

References 175