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Interconnected Networks

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Preface

Complex networks are powerful allies of our quest to tackle complexity in all of science. Many lines can be written about the benefits of using networks to study complex systems. Nevertheless, if I had to name their single most appealing property, I would say *simplicity*. One can map the interacting elements of any system to a set of nodes, and connect these nodes with a set of links according to their interactions. That is all it takes to build a network. Such a powerful abstraction allows to study many seemingly unrelated systems with a unified set of tools, and allows different scientific fields to benefit from advancements in other disciplines. Thus, it comes as no surprise that network science keeps growing in popularity.

However, while most early results about networks and their properties were obtained under the assumption that networks are isolated, in reality many networks interact with other networks. Consider, for example, our modern societies where individuals participate in different online social networks while they maintain a sizable amount of off-line contacts. These individuals are the means by which different social networks interact, so that information can propagate from one network to the others. Other examples include, but are not limited to, technological or infrastructure networks, whose proper function may depend on the function of another network, or transportation networks which are usually organized in layers that provide complementary access to different locations. In order to understand this bigger picture, it became clear recently that we have to extend our complex networks framework, and we are now able to treat such interconnected systems as *multilayered* networks.

In a multilayered representation each individual layer represents an isolated network from the set of networks that describe the whole system, as shown in Fig. 1. The presence of links between different networks (layers) can alter the way an interconnected system of networks behaves, even though this interconnectivity does not alter the basic characteristics of the individual networks in terms of function and topology (e.g., a communication network remains a communication network even though it is connected to the power grid). Note that the multilayered view is not just another way to describe communities in a single network, as it allows to describe systems with different types of interactions among and within the various

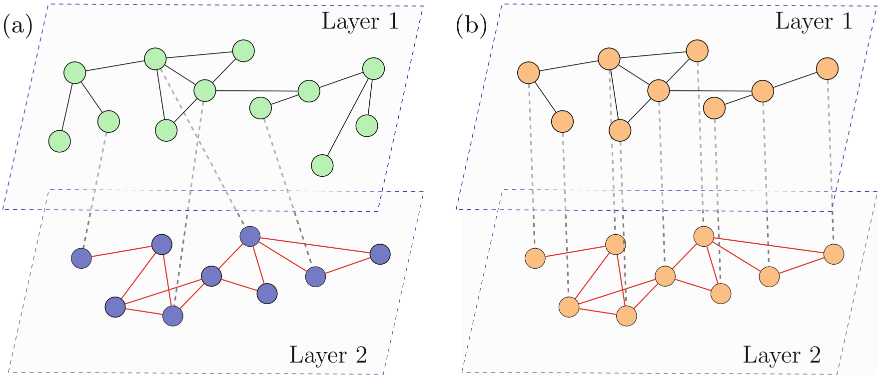

Fig. 1 Example of a two-layered network with different interconnectivity patterns. (**a**) The general case of a two-layered interconnected network where the two layers can have different number of nodes and not all nodes in one layer are interconnected to nodes of the other layer. (**b**) The special case of a multiplex network, where the nodes in the two layers are exactly the same. In this case there is a one-to-one connection between the nodes of both layers to represent their identity relation. Note, however, that in both cases, the links within the layers can be different, as indicated here with *red* and *black lines*

layers. Indeed, this nontrivial coupling allows for nonlinear effects and feedback loops, which generate emergent features that are visible only through the system as a whole and disappear when studying its individual component networks. Therefore, understanding the role of the links that connect individual layers (*interconnecting links*) and the way their presence affects the behavior of interconnected networks is a crucial step toward a more accurate description of real systems.

However, because of the different ways individual layers can be interconnected, and because interconnecting links may have different functions with respect to normal interlayer links, different naming schemes appeared aiming to distinguish cases of interest. But, instead of increasing clarity, names such as *interconnected networks*, *networks of networks*, *interdependent networks*, *multiplex networks*, etc. dominated the literature creating confusion about their actual meaning and proper use, especially with respect to what is different between them.

In order to clarify this subject, throughout this book we will call *interconnected* multilayered (or multilayer) networks the general case where there is no particular assumption with respect to the connectivity patterns and/or the function of the *interconnecting links*. Networks with such general connectivity structures are also called *interacting networks* or *networks of networks* in the literature. The special cases where the same set of nodes appear across different layers while the links within the layers are different are called *multiplex networks*. Multiplex networks are useful to describe different categorical relationships between nodes, like a set of people communicating via different channels (like phone calls, emails, etc.).

In the general case interconnecting links provide the means of interaction between networks. But if the functional properties of these links induce dependency relations, so that nodes from one layer depend on nodes form other layers to function properly, then the system of networks is called *interdependent network*. Such networks are very important, especially when studying critical infrastructures and systemic risk, because nodes that may seem safe from the single network perspective may have exposures via dependency links to other networks, which make them extremely fragile. Of course, various combinations of network-to-network connectivities with different functional properties of interconnecting links are allowed. Thus, one may encounter multiplex networks with dependency links, or networks with partial dependency links where one layer depends on another and not vice versa, etc.

With this volume we want to provide a collection of works that highlight and summarize recent developments on network theory, signaling the emergence of a mathematical, computational, and algorithmic framework that deals with interconnected complex networks, both on a theoretical and practical level. Individual chapters deal with related but in most cases complementary subjects. Each chapter is self-contained and can stand on its own. For the interested reader this removes the need to follow a particular order and allows to focus on specific subjects. However, the structure of the book follows, indeed, a specific pattern, starting with the more theoretical works and gradually dealing with more practical subjects and applications.

More precisely, the book consists of the following chapters:

- Chapter 1 shows that the formation of interconnected networks undergoes a structurally sharp (discontinuous) transition, depending on the relative importance of the links within and across layers.
- Chapter 2 describes the topology of an interconnected system of networks in terms of matrices and discusses about several metrics that are key to characterize multilayer networks and their spectral properties.
- Chapter 3 investigates diffusion dynamics on multilayer networks when we have incomplete knowledge about the link formations inside or across the layers, using ensembles of interconnected networks with similar characteristics.
- Chapter 4 describes how choosing the adequate connector links between networks may promote or hinder different structural and dynamical properties of a particular network.
- Chapter 5 provides a review of recent advances on the role of connectivity and dependency links in the robustness of interconnected networks, focusing on the dynamics of cascading failures on interdependent networks.
- Chapter 6 uses percolation theory to describe damage resilience of interconnected (multiplex) networks, following two alternative definitions on the pruning process that alter the nature of the percolation transition.
- Chapter 7 explores how much interconnectivity is needed for the emergence of cooperation in interconnected networks and shows that an intermediate density of sufficiently strong interactions between networks is the optimal case.

- Chapter 8 analyzes the influence of a time delay on a system of two interconnected networks of oscillators and explores its dynamics as a function of the couplings and communication lag.
- Chapter 9 deals with the architecture of real urban mobility networks from the multiplex network's perspective using empirical data of mobility patterns in two cities. This reveals that the socioeconomic characteristics of the population have an extraordinary impact on the layer organization of these systems.
- Chapter 10 provides a new understanding of the social structure of elites by analyzing the community structure of the generalized K-core and by identifying weakly connected regions that bridge core communities on a multiplex system, using data from a Massive Multiplayer Online Game.
- Chapter 11 reviews the empirical structure of the multiplex interbank networks and the theoretical consequences of this representation using Maximum Entropy null models.
- Chapter 12 describes the phenomenology of multilevel financial networks by reviewing selected theoretical and empirical works providing arguments in favor of adopting the broad view of the network approach to finance.

Closing this short introduction, I would like to thank all authors for their contributions and for their fruitful collaboration. Even though there are much more to be discussed about interconnected multilayered networks than what is covered in this volume, I do believe that the reader will find this collection both inspiring and motivating. I would also like to thank Frank Schweitzer for his valuable guidance that made this book possible and to acknowledge support from the EU FET project MULTIPLEX 317532.

Zürich, Switzerland Antonios Garas
2015

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